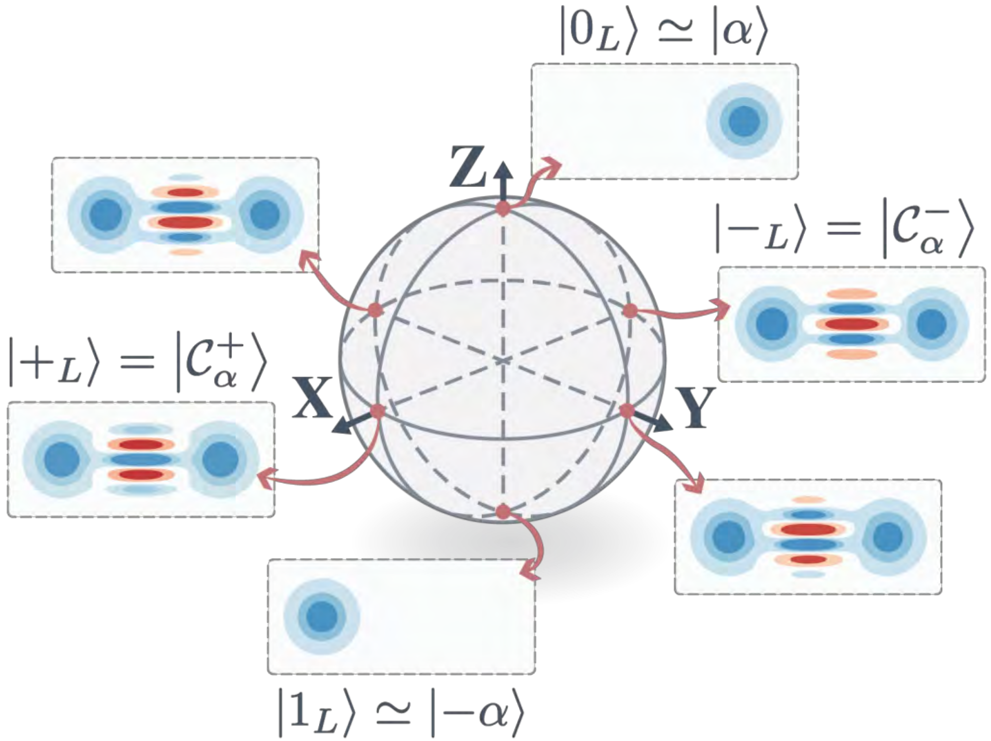
ALICE & BOB

Fault-tolerant quantum computing against phase-flip noise

Q-FEEDBACK WORKSHOP | JÉRÉMIE GUILLAUD

JUNE 2026

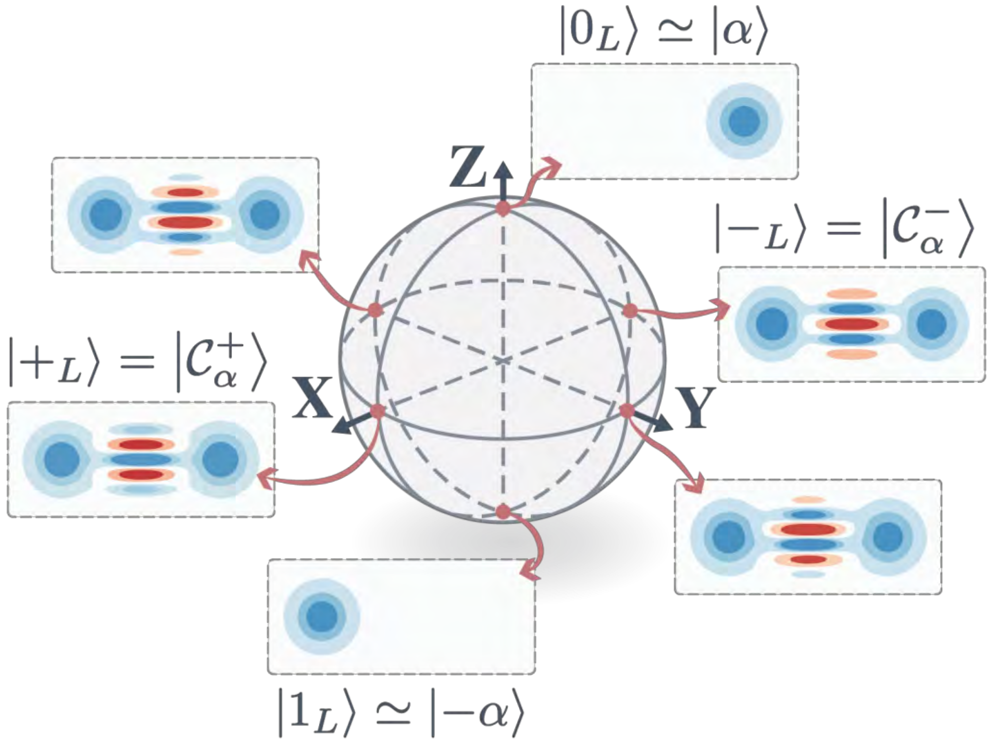
Cat qubit encoding



$$|\mathcal{C}_\alpha^+\rangle \doteq \mathcal{N}_+(|\alpha\rangle + |-\alpha\rangle) \propto \sum \frac{\alpha^{2n}}{\sqrt{(2n)!}}|2n\rangle$$

$$|\mathcal{C}_\alpha^-\rangle \doteq \mathcal{N}_-(|\alpha\rangle - |-\alpha\rangle) \propto \sum \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle$$

Cat qubit stabilization



$$\frac{d\rho}{dt} = -i[H,\rho] + \Sigma_k L_k \rho L_k^\dagger + \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

« Dissipative cat qubit »

$$L = \sqrt{\kappa_2}(a^2 - \alpha^2)$$

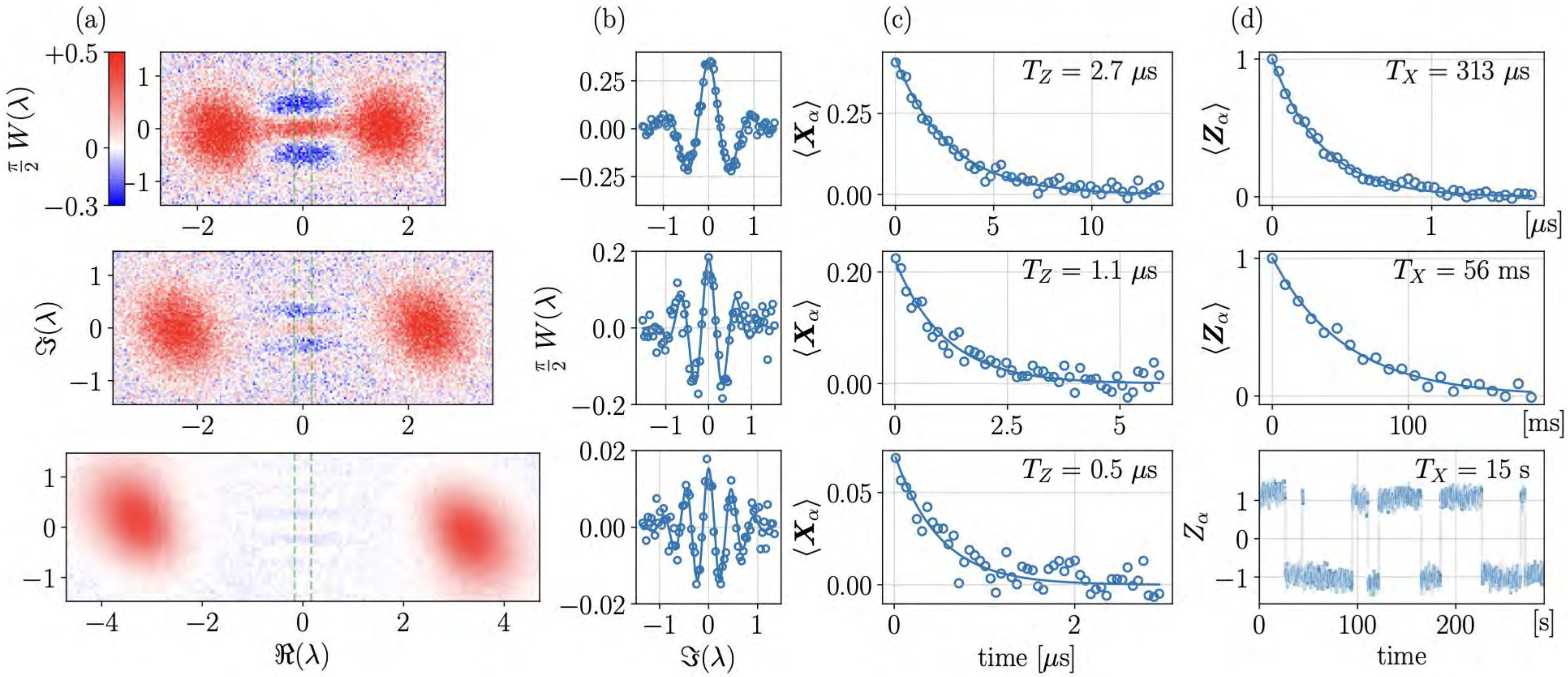
M. Mirrahimi *et al, New J. Phys.* **16** 045014

« Kerr cat qubit »

$$H = -K(a^2 - \alpha^2)^\dagger (a^2 - \alpha^2)$$

S. Puri *et al, npj Quantum Information 3 (1), 1-7*

Exponential suppression of bit-flips



$\alpha^2 = 2.5$

$\alpha^2 = 5.6$

$\alpha^2 = 11.3$

U. Réglade, A. Bocquet et. al, Nature 629, 778-783 (2024)

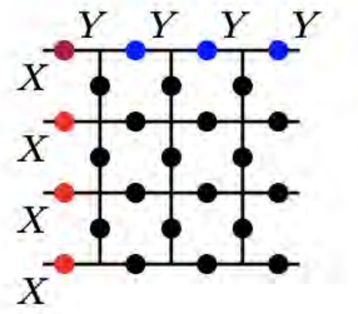
01

BUILDING A COMPACT MEMORY

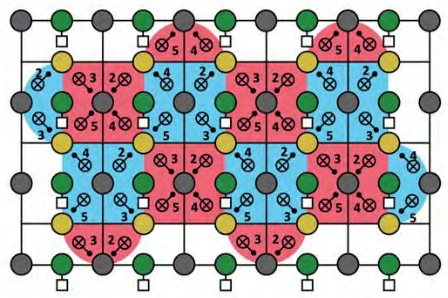
Fully protected logical qubit?

Include some bit-flip error correction capability ? → bias-tailored codes

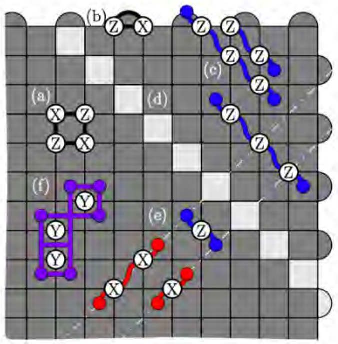
D. Tuckett *et al,* PRL (2018)



C. Chamberland et al, PRX Quantum (2022)

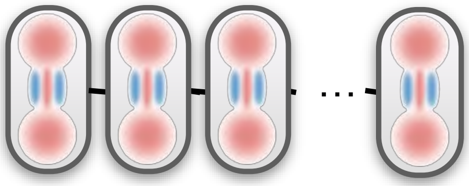


J. Pablo Bonilla Ataides *et al,* Nature Com. (2021)

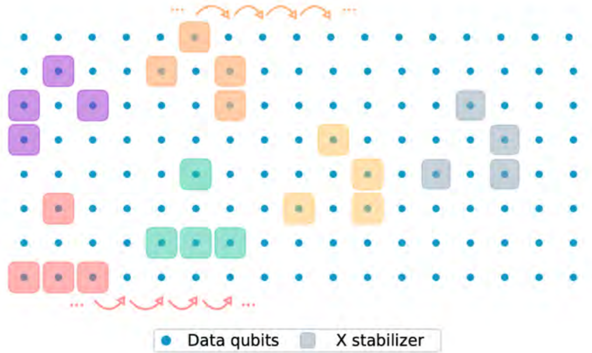


$\bar{n} = |\alpha|^2$ **large enough to handle bit-flips alone ?** → phase-flip codes (« classical » codes)

JG and M. Mirrahimi, PRX (2019)



D. Ruiz et al, Nature Com. (2025)



Concatenating cat qubits in high-rate phase-flip codes

LDPC-cat codes for low-ov

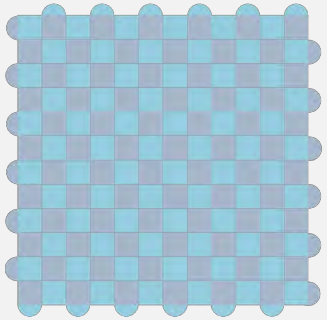
Diego Ruiz,[1,2,*] Jérémie Guillaud,[1] Anthony [4]

	Surface code + sc qubits [1, 2]	H cod	ate LDPC cat qubits
Short-range interactions Tanner graph degree	yes (2D) 3-4		s (2D) 4
$N_L = 100$ footprints $\left.\begin{array}{l}\epsilon = 10^{-3}\\ \kappa_1/\kappa_2 = 10^{-4}\end{array}\right\} \rightarrow \epsilon_L \leq 10^{-8}$	N = 33,700 -		- (N/44)

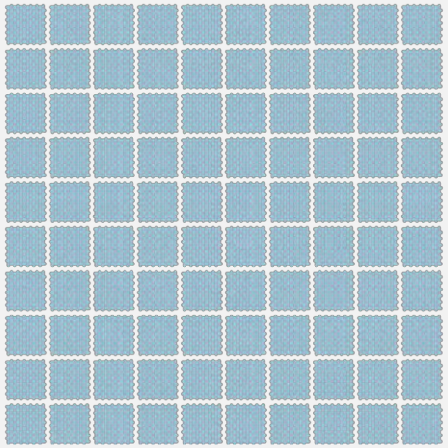
$$\epsilon_L \approx 0.1(100p)^{(d+1)/2}$$

$$d = 13,$$
$$p = 10^{-3} \rightarrow \epsilon_L = 10^{-8}$$

$$N_{sc} = 2d^2 - 1 = 337$$



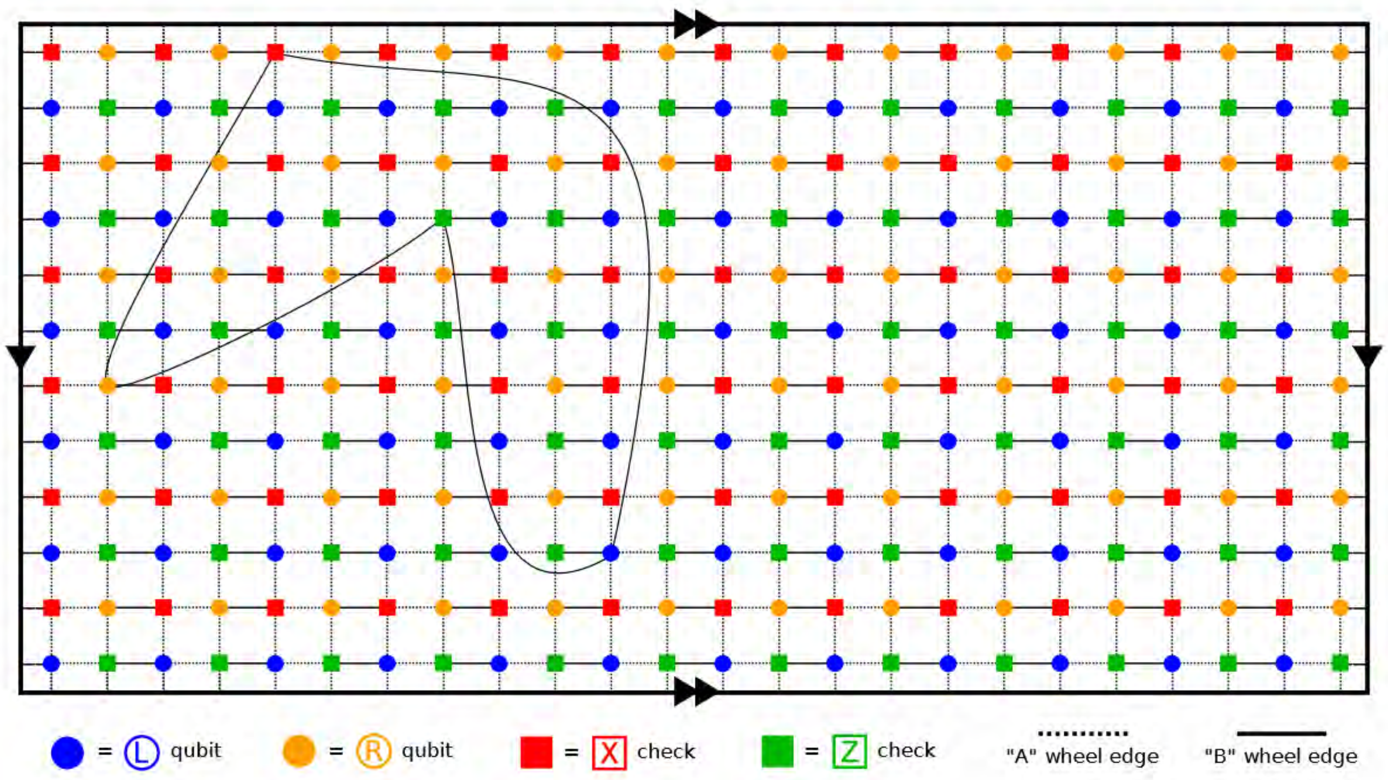
$$N = 100N_{sc} = 33{,}700$$



Nature Communications 16, 1040 (2025)

Bravyi Poulin Terhal bound for 2D local quantum codes

<u>For a quantum local 2D code</u> : $kd^2 \leq O(n)$
Surface code family: $k = 1, d = \sqrt{n}$



S. Bravyi et al., PRL 104 (2010)
S. Bravyi et al., Nature 627 (2024)

Bravyi Poulin Terhal bound for 2D local phase-flip codes

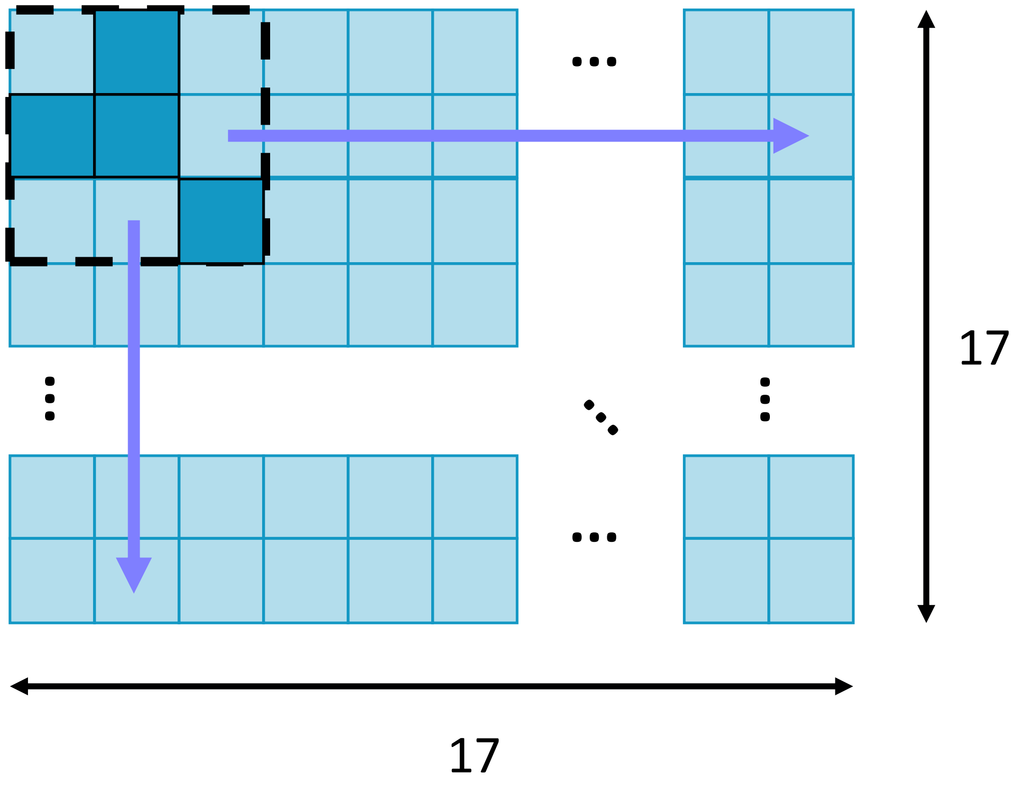
<u>For a classical local 2D code</u> : $k\sqrt{d} \leq O(n)$

Repetition code family : $k = 1, d = n$

A code could have $k = O(\sqrt{n})$ and $d = O(n)$

Distance as good as repetition code but encodes $\sqrt{n}$ qubits

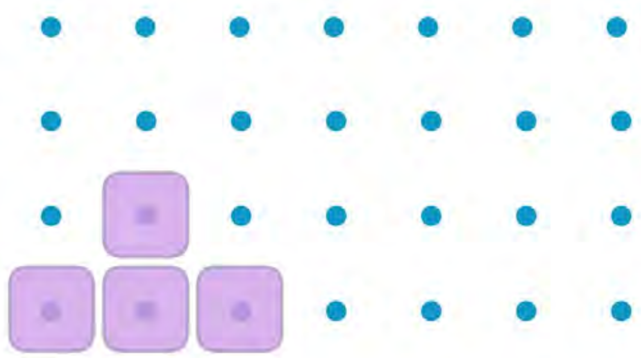
<u>Brute force search</u>

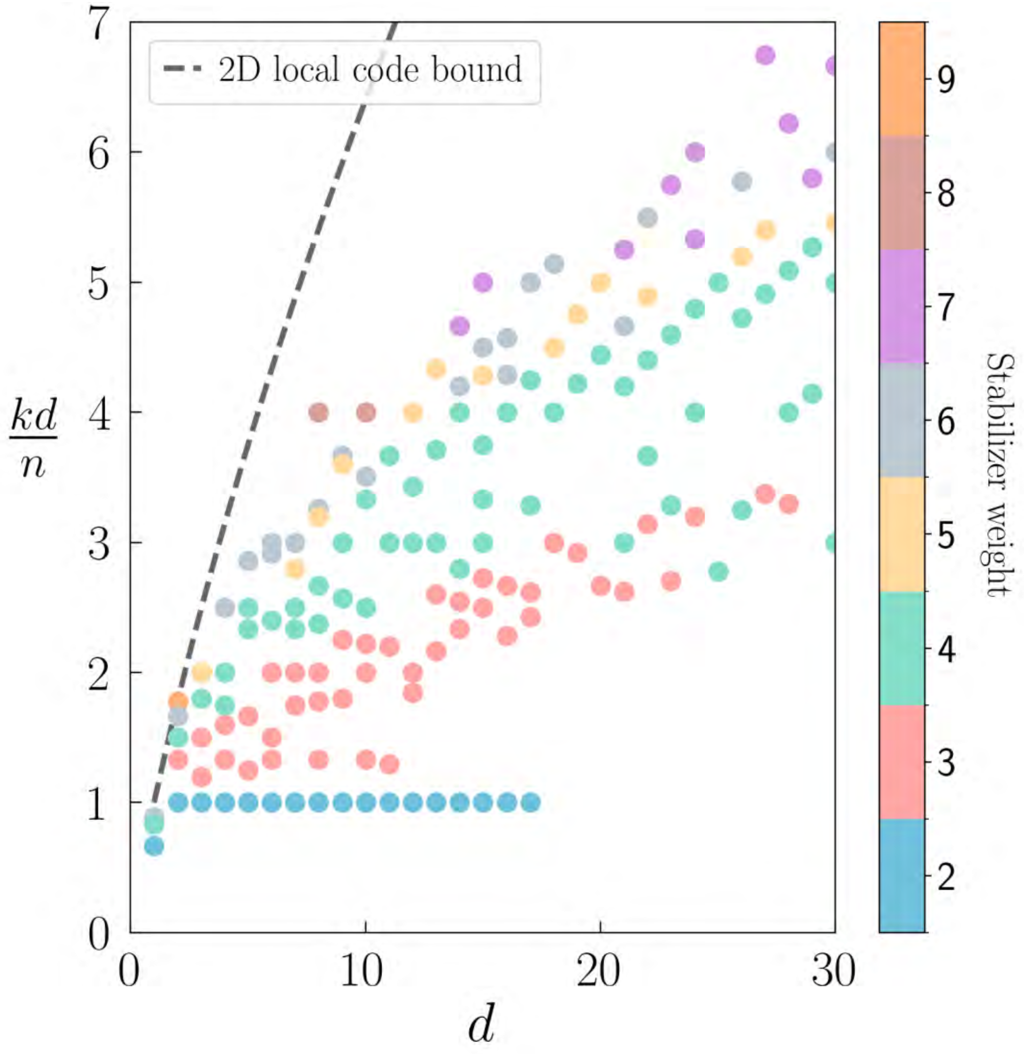


S. Bravyi et al., PRL 104 (2010)

2D local phase-flip codes

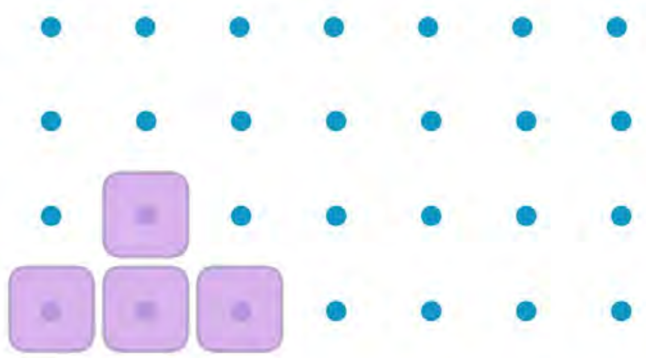
Single stabilizer shape (translated)



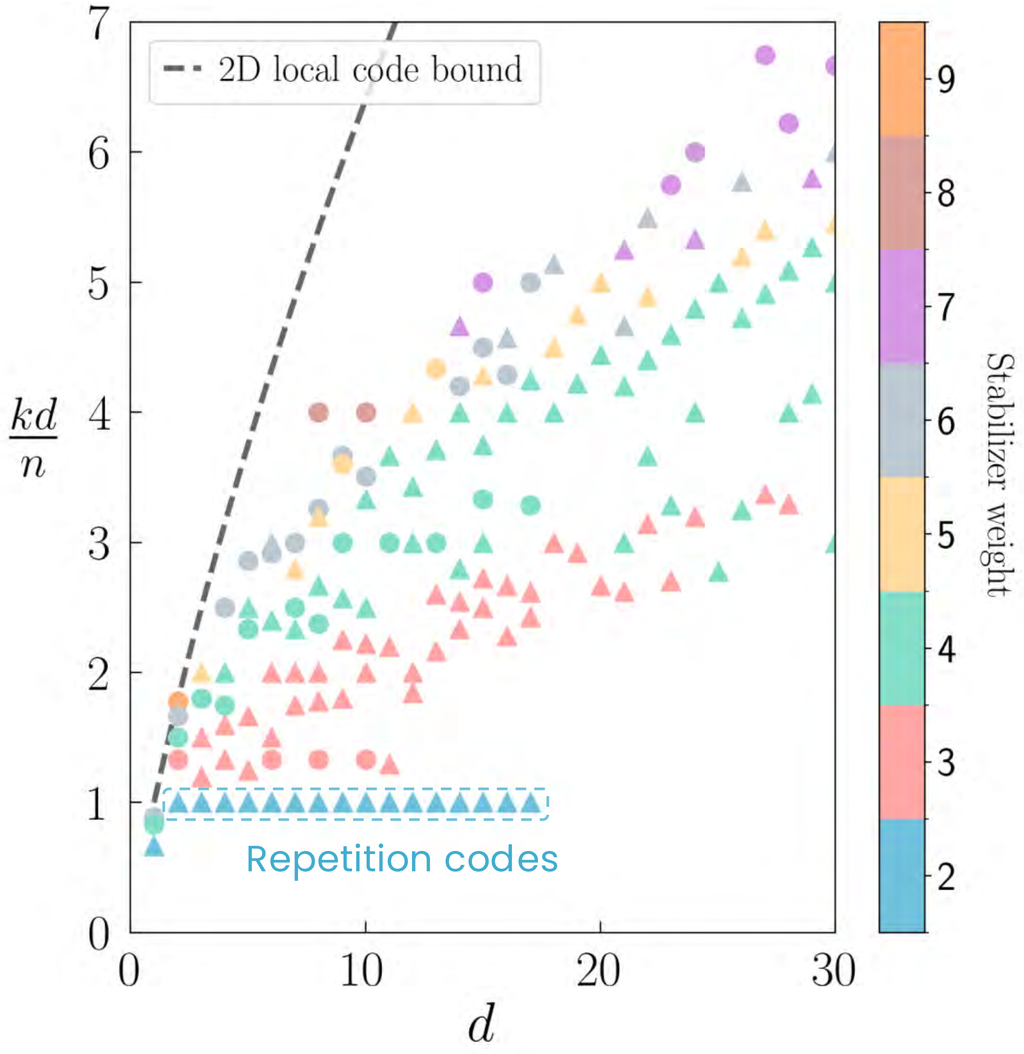


2D local phase-flip codes

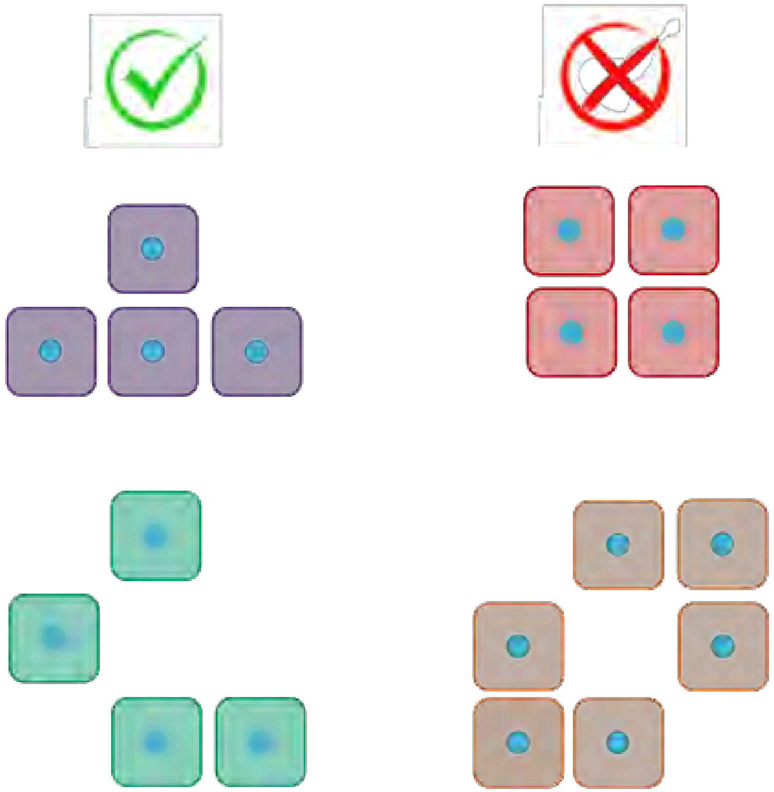
Single stabilizer shape (translated)



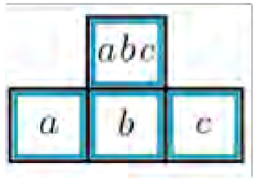
▲ Cellular automaton codes

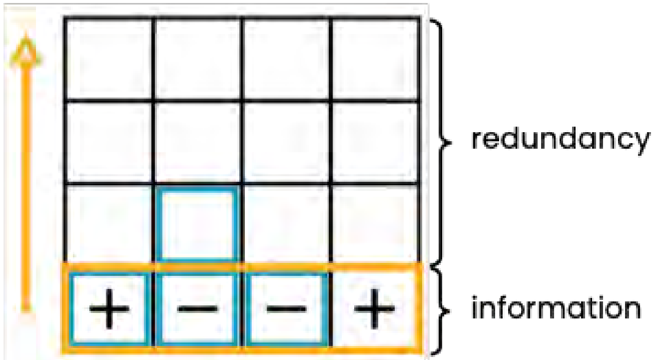


Cellular automaton (phase-flip) codes



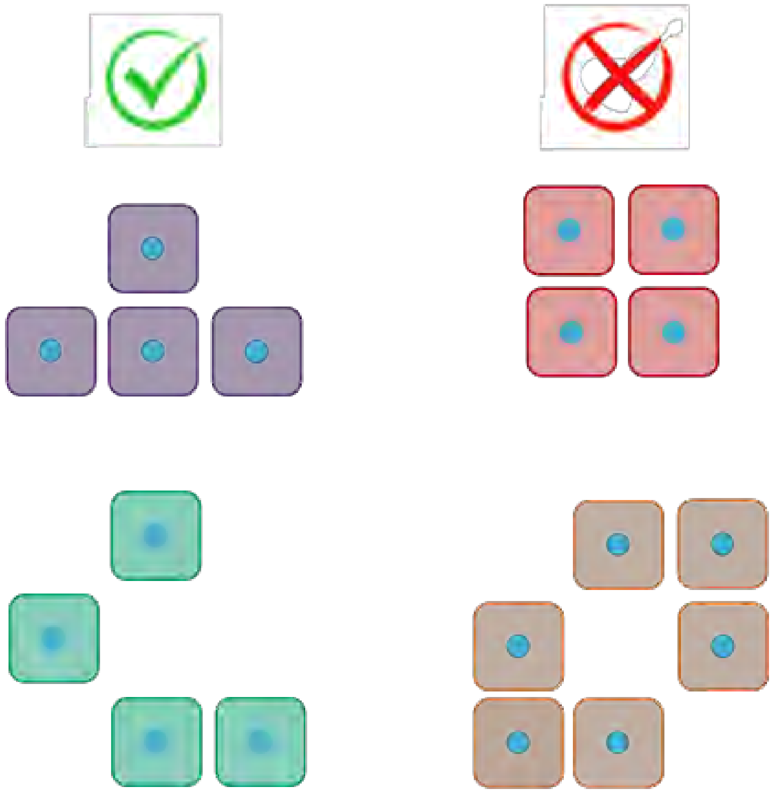
Cellular automaton rule to construct the codewords:





S. Bravyi et al, PRL 104, 2010
M. Newman et al, PRE 60, 1999
G. M. Nixon et al, IEEE 67, 2021

Cellular automaton (phase-flip) codes



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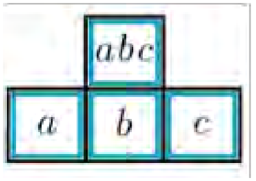
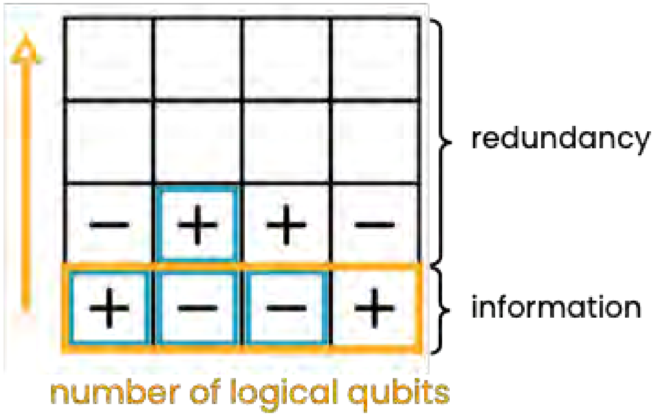
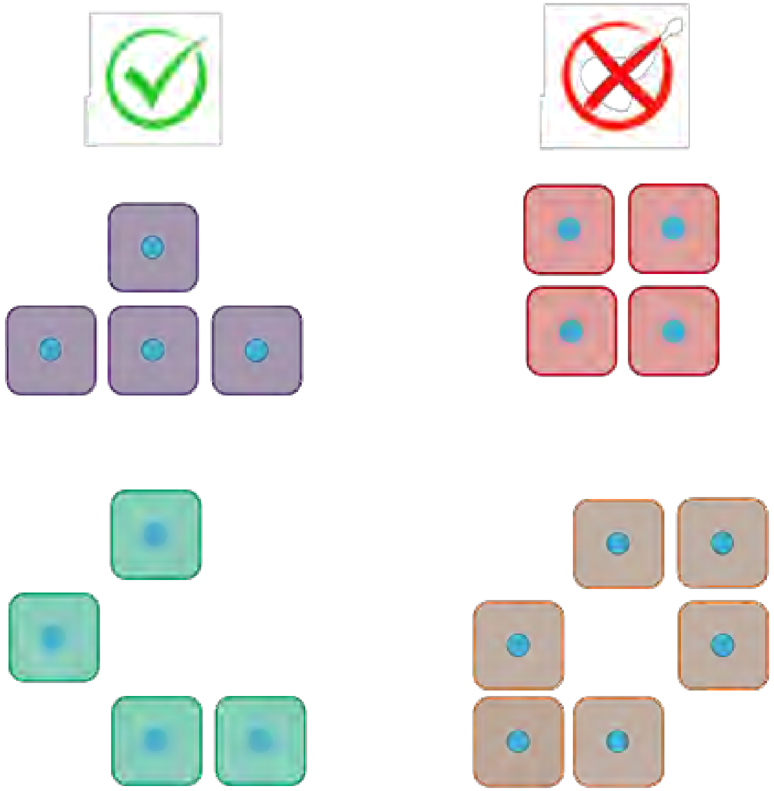




S. Bravyi et al, PRL 104, 2010
M. Newman et al, PRE 60, 1999
G. M. Nixon et al, IEEE 67, 2021

Cellular automaton (phase-flip) codes



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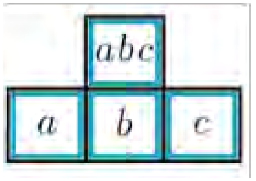
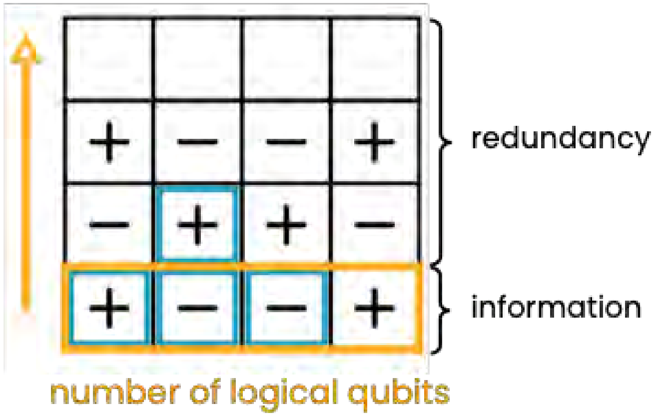
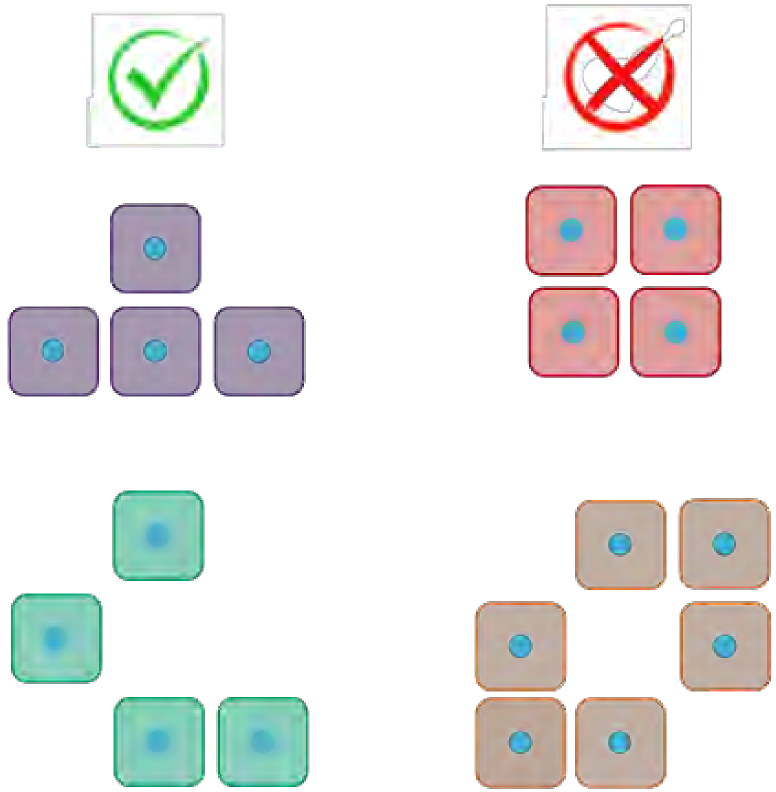


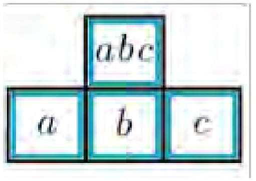


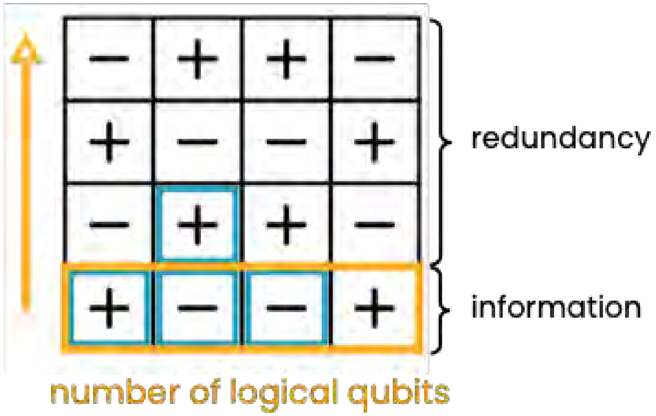
S. Bravyi et al, PRL 104, 2010
M. Newman et al, PRE 60, 1999
G. M. Nixon et al, IEEE 67, 2021

Cellular automaton (phase-flip) codes



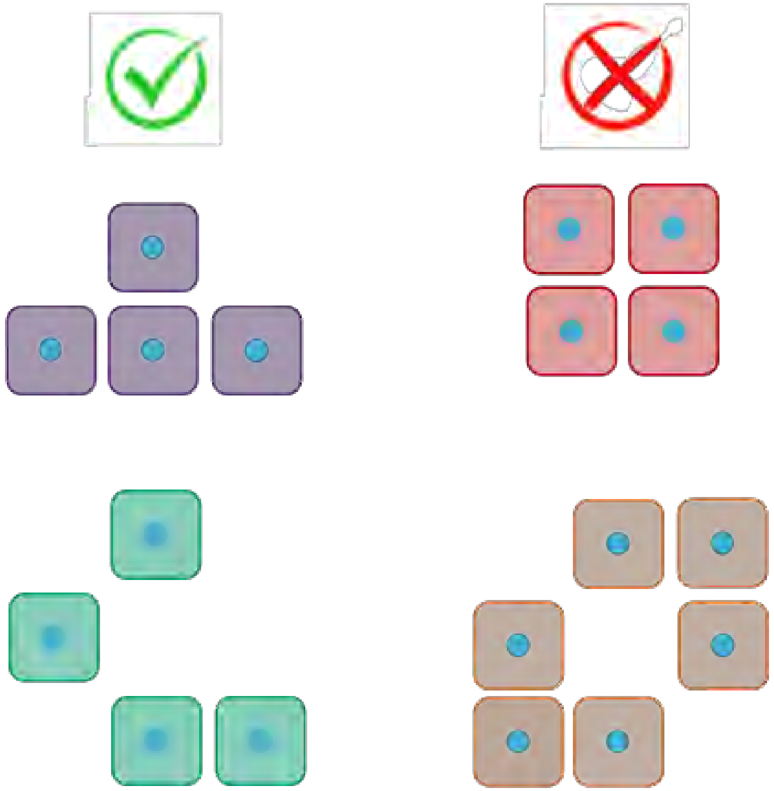
Cellular automaton rule to construct the codewords:



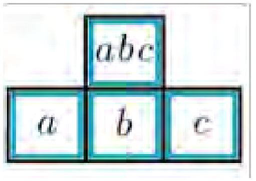


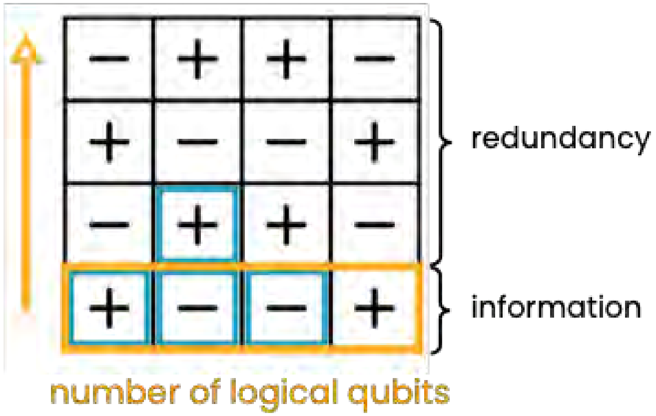
S. Bravyi et al, PRL 104, 2010
M. Newman et al, PRE 60, 1999
G. M. Nixon et al, IEEE 67, 2021

Cellular automaton (phase-flip) codes



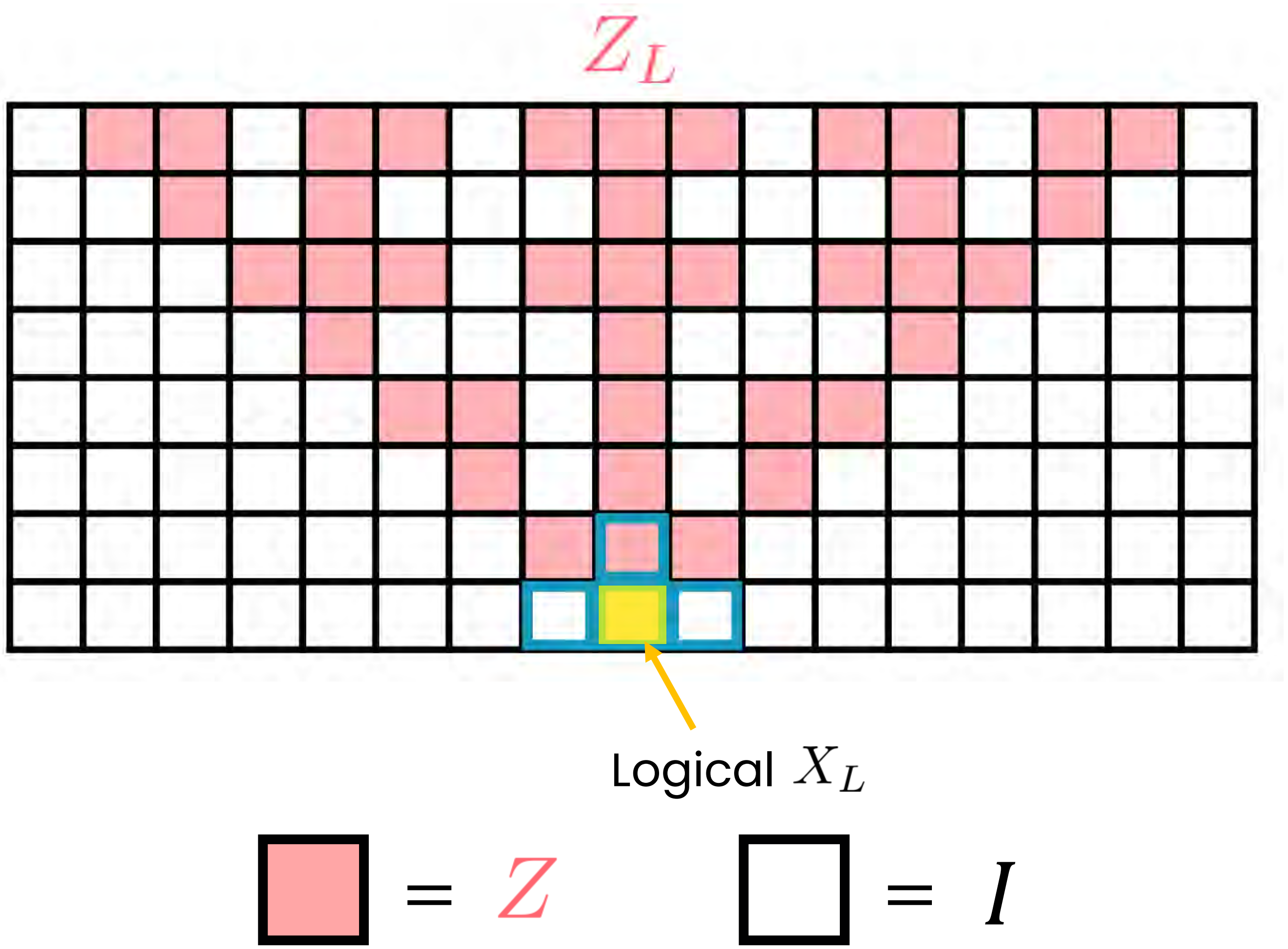
Cellular automaton rule to construct the codewords:





S. Bravyi et al, PRL 104, 2010
M. Newman et al, PRE 60, 1999
G. M. Nixon et al, IEEE 67, 2021

Fractal (phase-flip) codes



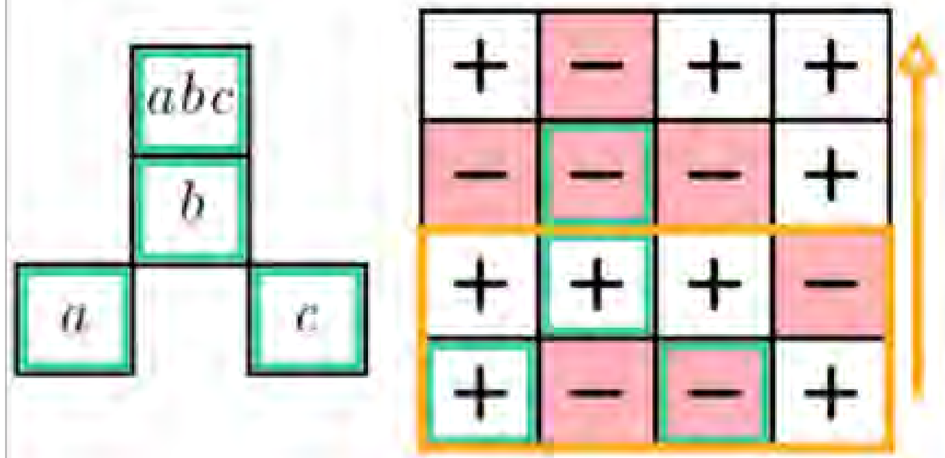
Minimum weight logical Z_L operator with fractal support

Basis of logical operator can be formed by horizontal translation

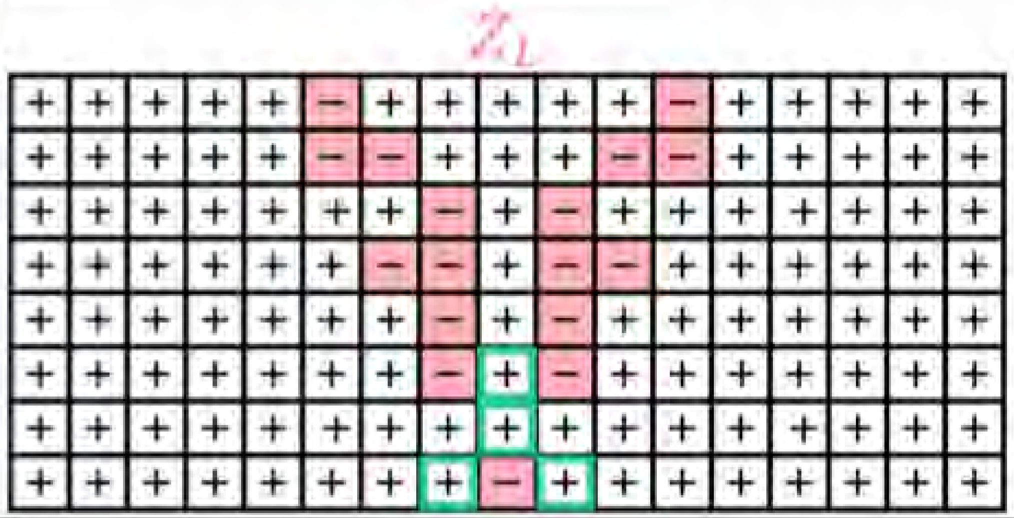
Logical X_L of weight one → bit-flip correction handled by cat qubits

B. Yoshida, Annals of Physics 338:134-166, 2013

Optimising cellular automaton codes (1/2)



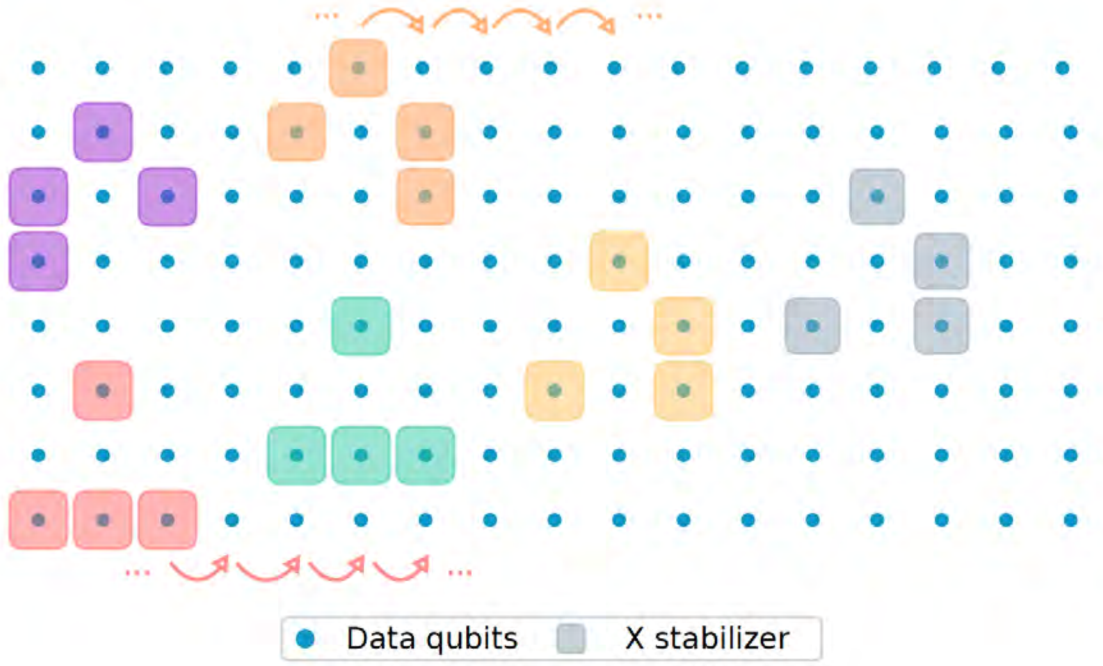
Double the number of logical qubits

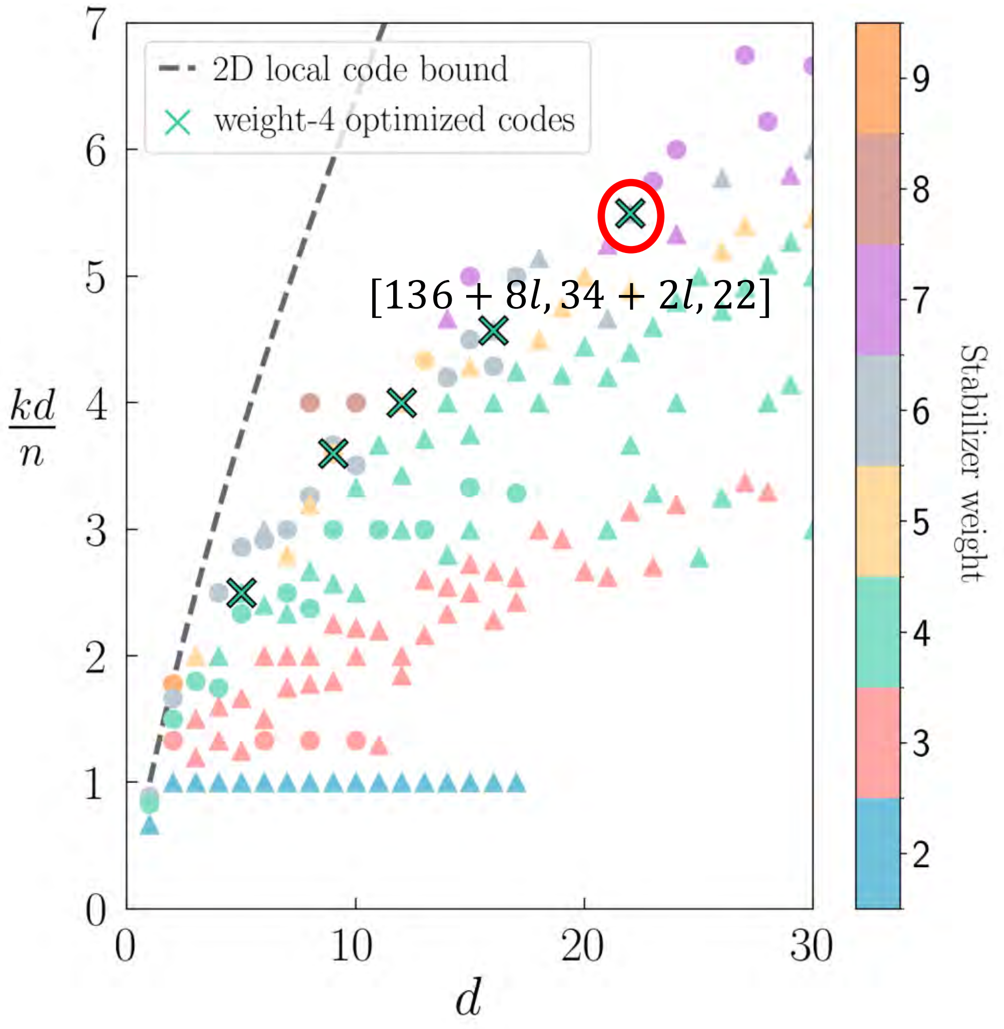


Reduce the distance

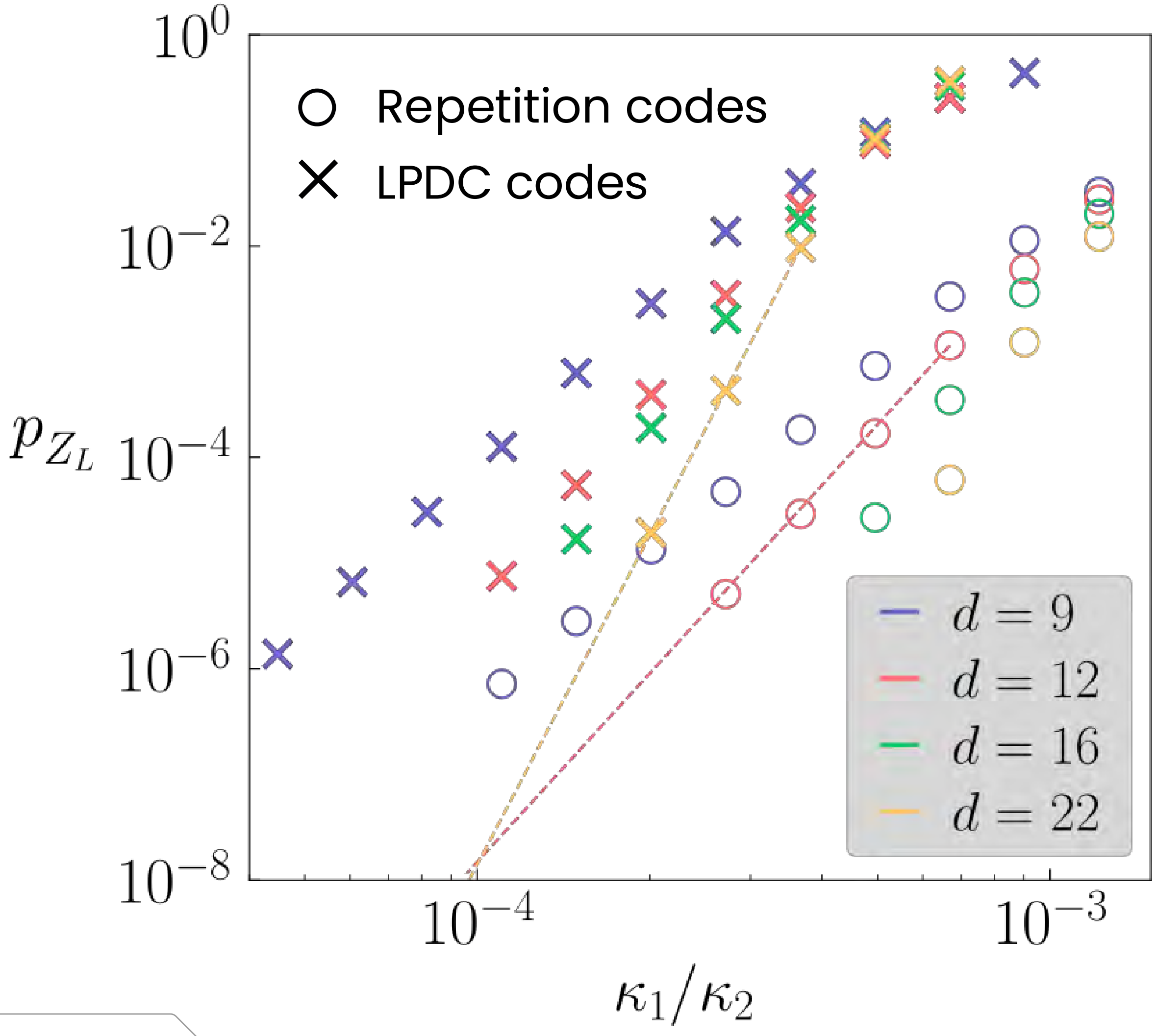
Overall, the overhead reduction factor kd/n is improved

Optimising cellular automaton codes (2/2)

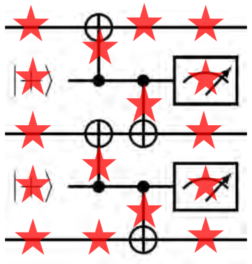




(Numerical) Performance under circuit-level noise



Circuit-level noise model

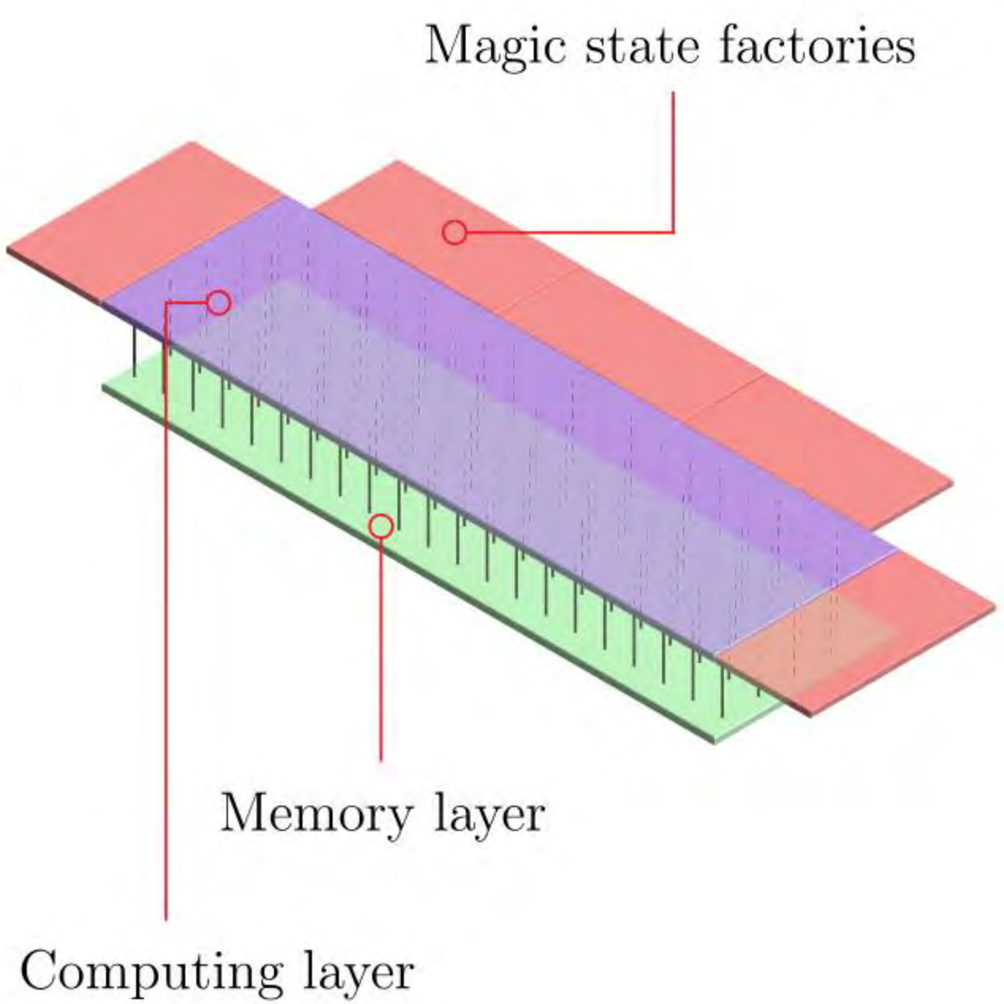


BP+OSD decoder quantumgizmos/bp_osd

02

(FTQ) COMPUTING ON “PHASE-FLIP CODE + CAT QUBITS”

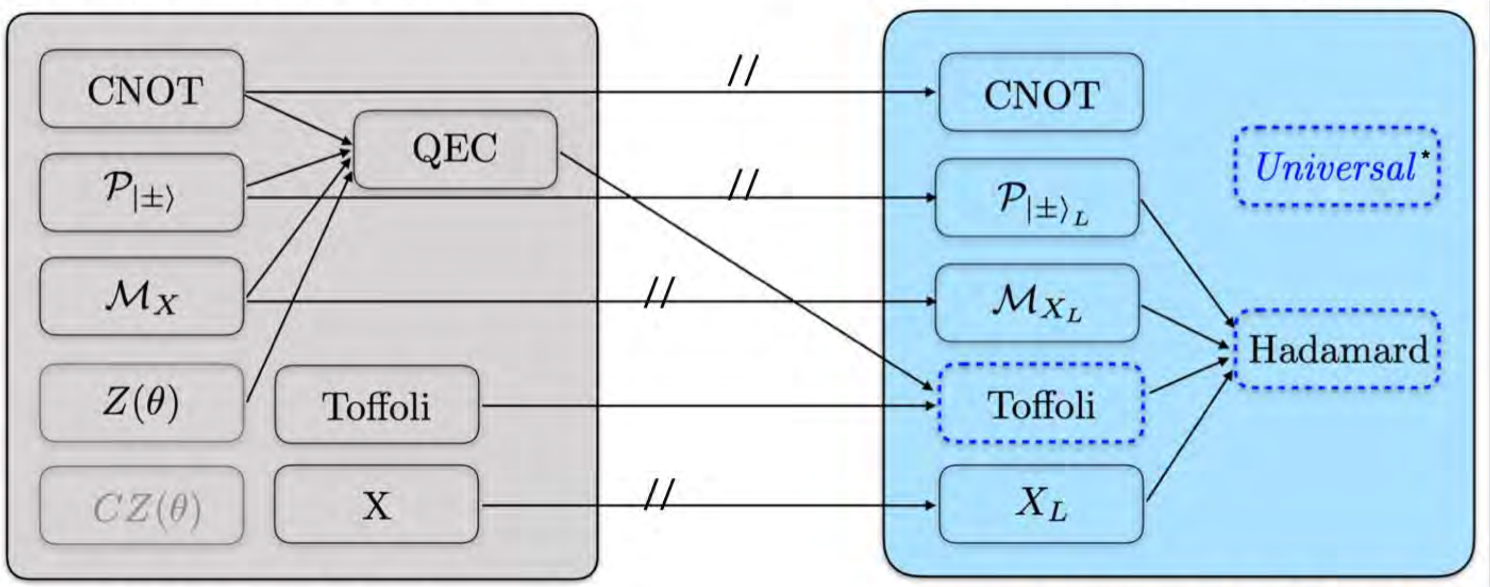
Architecture (artist view)



Universal set of logical gates

« Bias-preserving » operations

Fault-tolerant logical operations



Single cat-qubit level

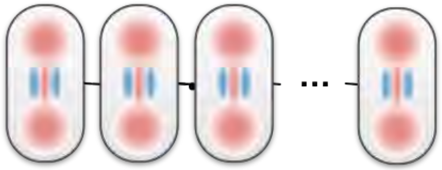
Repetition cat-qubit level

$$|+\rangle = |\mathcal{C}_\alpha^+\rangle$$

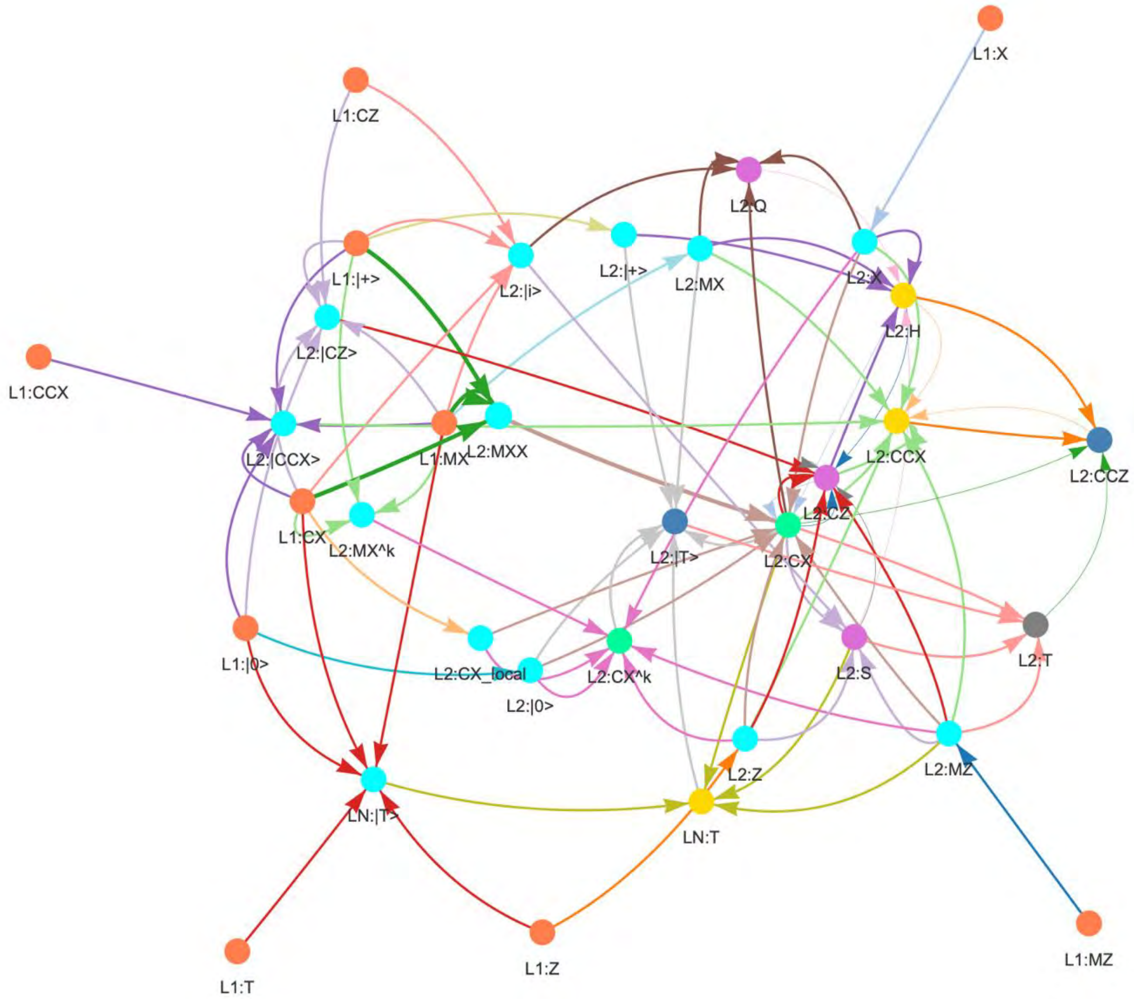
$$|-\rangle = |\mathcal{C}_\alpha^-\rangle$$

$$|+\rangle_L = |\mathcal{C}_\alpha^+\rangle^{\otimes n}$$

$$|-\rangle_L = |\mathcal{C}_\alpha^-\rangle^{\otimes n}$$



Universal set of logical gates



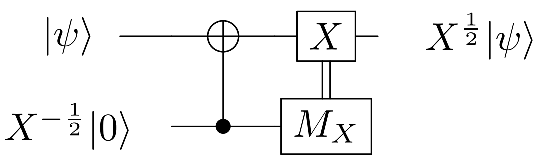
Universal set of logical gates

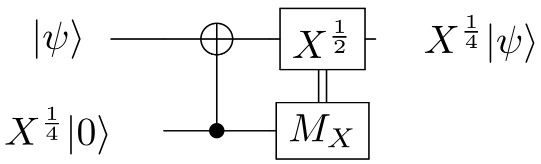
$$\{|0\rangle_L, M_Z, H_L, CNOT_L, S_L = Z_L^{1/2}, T_L = Z_L^{1/4}\}$$

$$\{|0\rangle_L, M_Z, H_L, CNOT_L, X_L^{1/2}, X_L^{1/4}\}$$

$$\{|X^{-1/2}\rangle_L = X_L^{-1/2}|0\rangle_L, CNOT_L, X_L, M_{X_L}\}$$

$$\{|X^{1/4}\rangle_L = X_L^{1/4}|0\rangle_L, CNOT_L, X_L^{1/2}, M_{X_L}\}$$



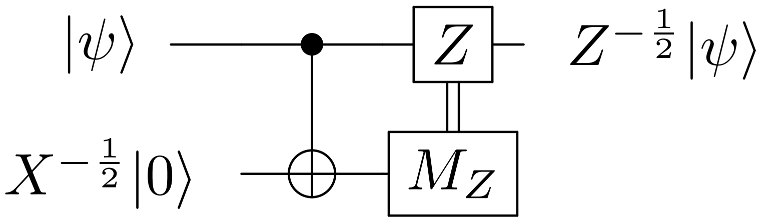


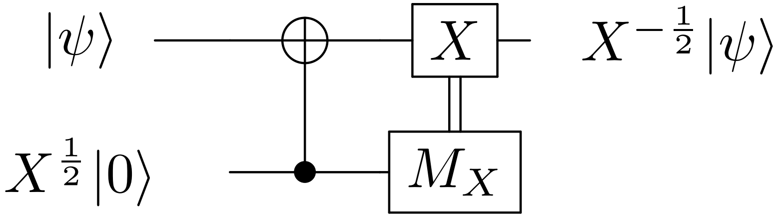
Compiling the Hadamard gate

$$H_L = Z_L^{-1/2} X_L^{-1/2} Z_L^{-1/2}$$

$$\{|X^{-1/2}\rangle_L = X_L^{-1/2}|0\rangle_L, CNOT_L, Z_L, M_{X_L}\}$$

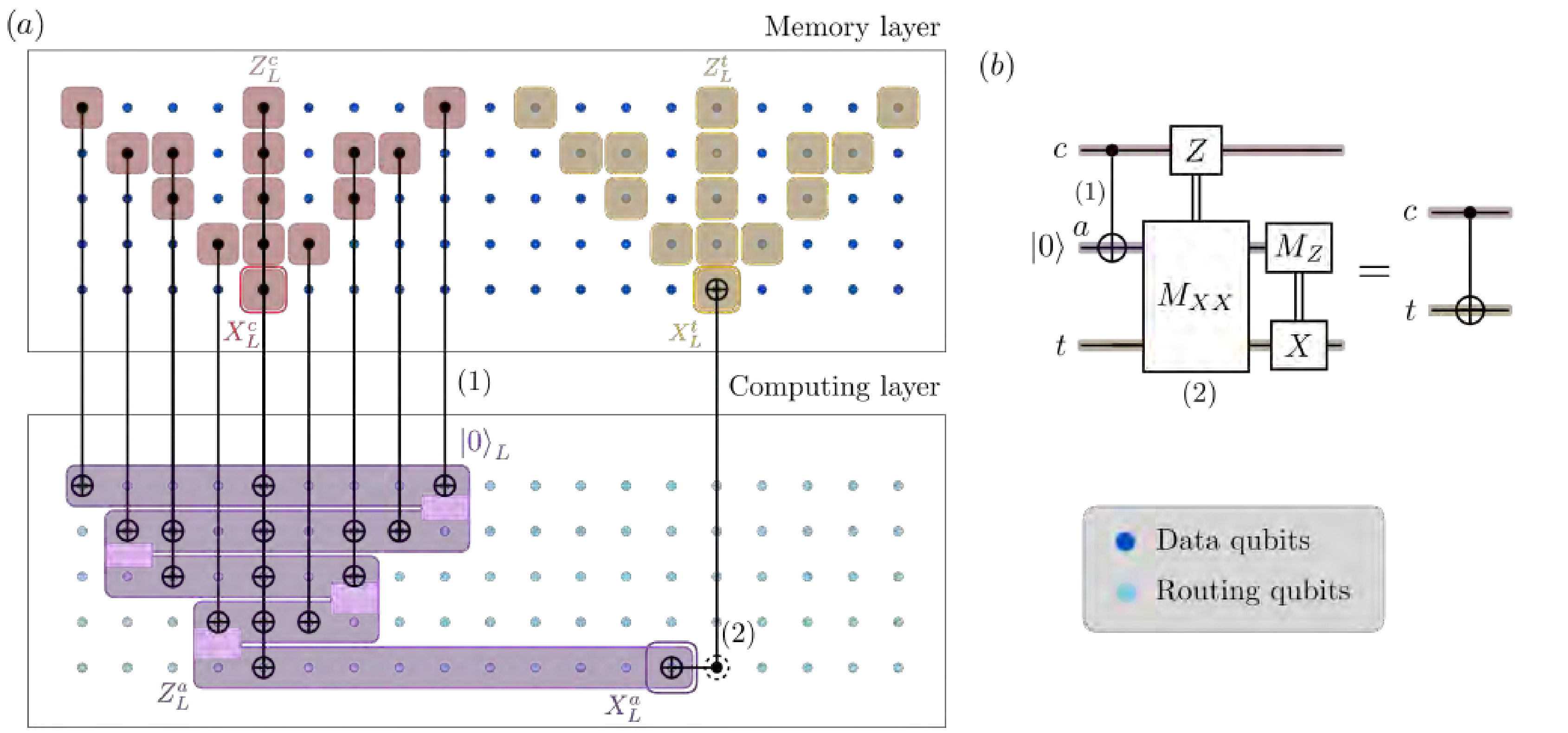
$$\{|X^{1/2}\rangle_L = X_L^{1/2}|0\rangle_L, CNOT_L, X_L, M_{X_L}\}$$





Compiling the (long-range) CNOT gate

$$CNOT_L \longrightarrow \{|0\rangle_L, CNOT_L^{||}, X_L, Z_L, M_{X_L X_L}, M_{Z_L}\}$$



Logical gates from physical gates

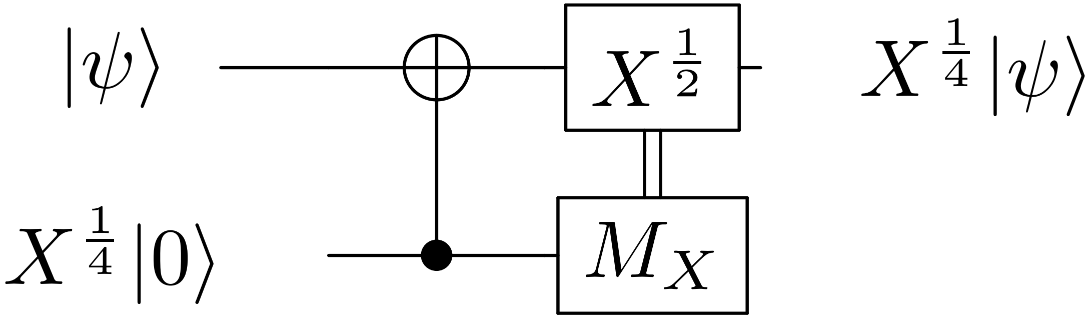
	$\vert 0\rangle_L$	$\vert X^{\pm 1/2}\rangle_L$	$\vert X^{1/4}\rangle_L$	X_L	Z_L	$CNOT_L^{\vert\vert}$	M_{X_L}	$M_{X_L X_L}$	M_{Z_L}
$\vert 0\rangle$	x	x	x						
$\vert +\rangle$	x	x	x					x	
Z					x				
X				x					
$X^{\pm 1/2}$		x							
$X^{\pm 1/4}$			x						
$CNOT$	x	x	x					x	
$CNOT^{\vert\vert}$						x			
M_X	x	x	x				x	x	
M_Z		x	x						x

03

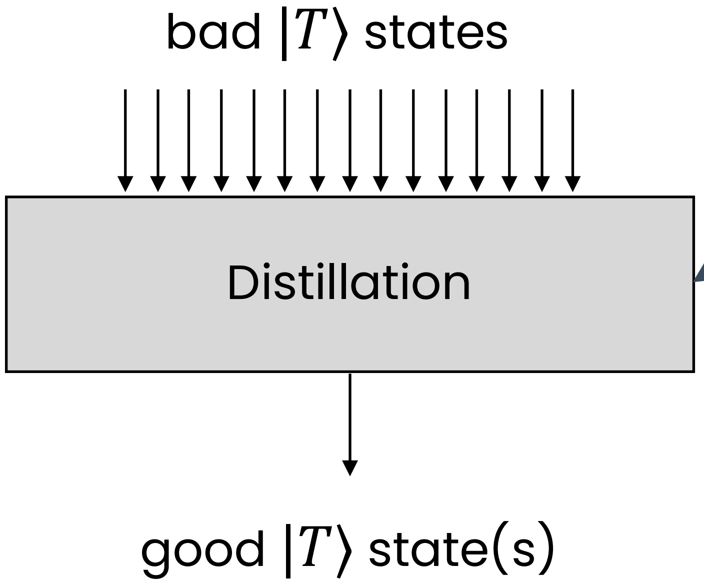
EFFICIENT STATE DISTILLATION WITH CAT QUBITS

Teleportation of a T gate

$$\{|X^{1/4}\rangle_L = X_L^{1/4}|0\rangle_L, CNOT_L, X_L^{1/2}, M_{X_L}\}$$



Distillation of a T state

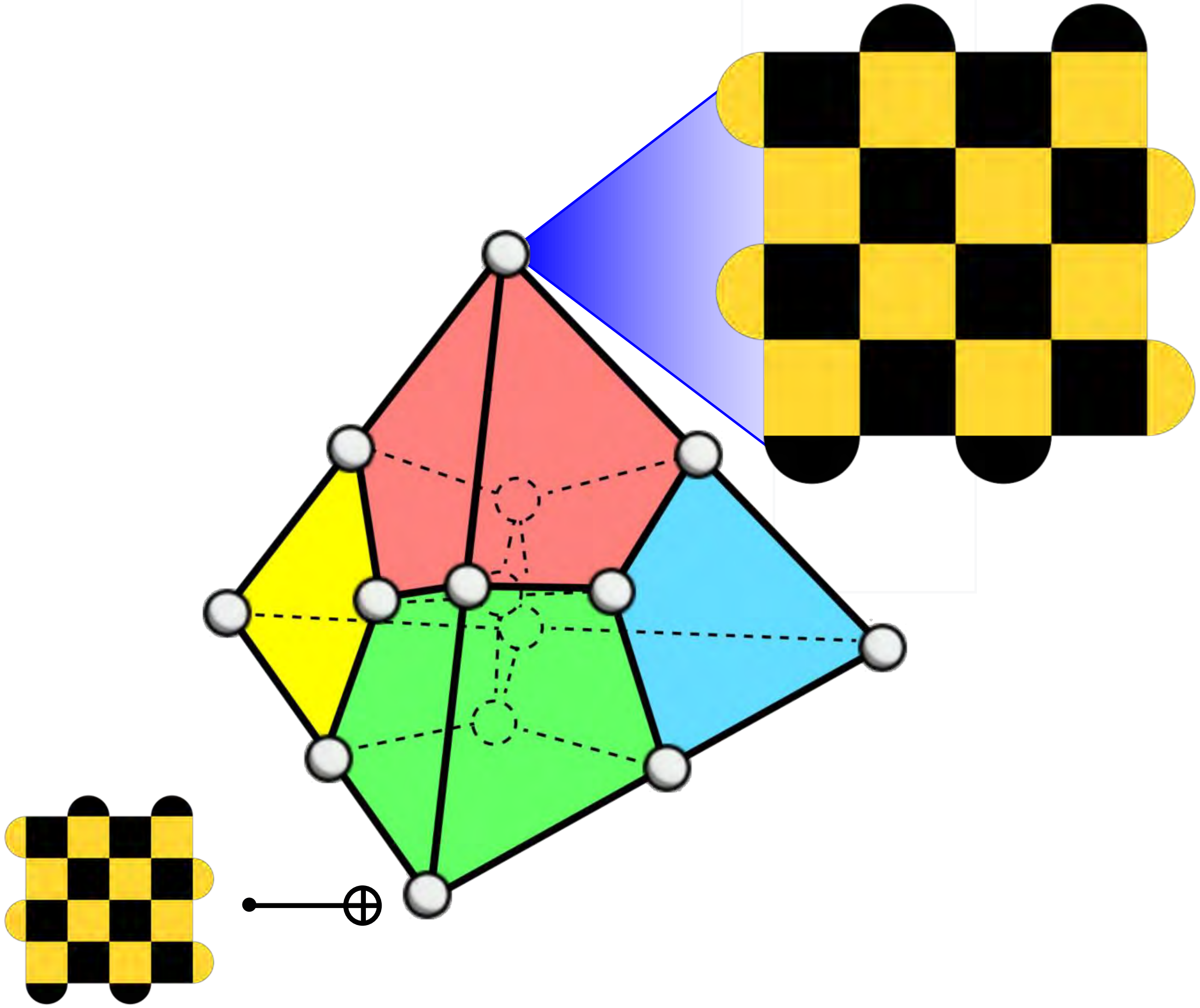


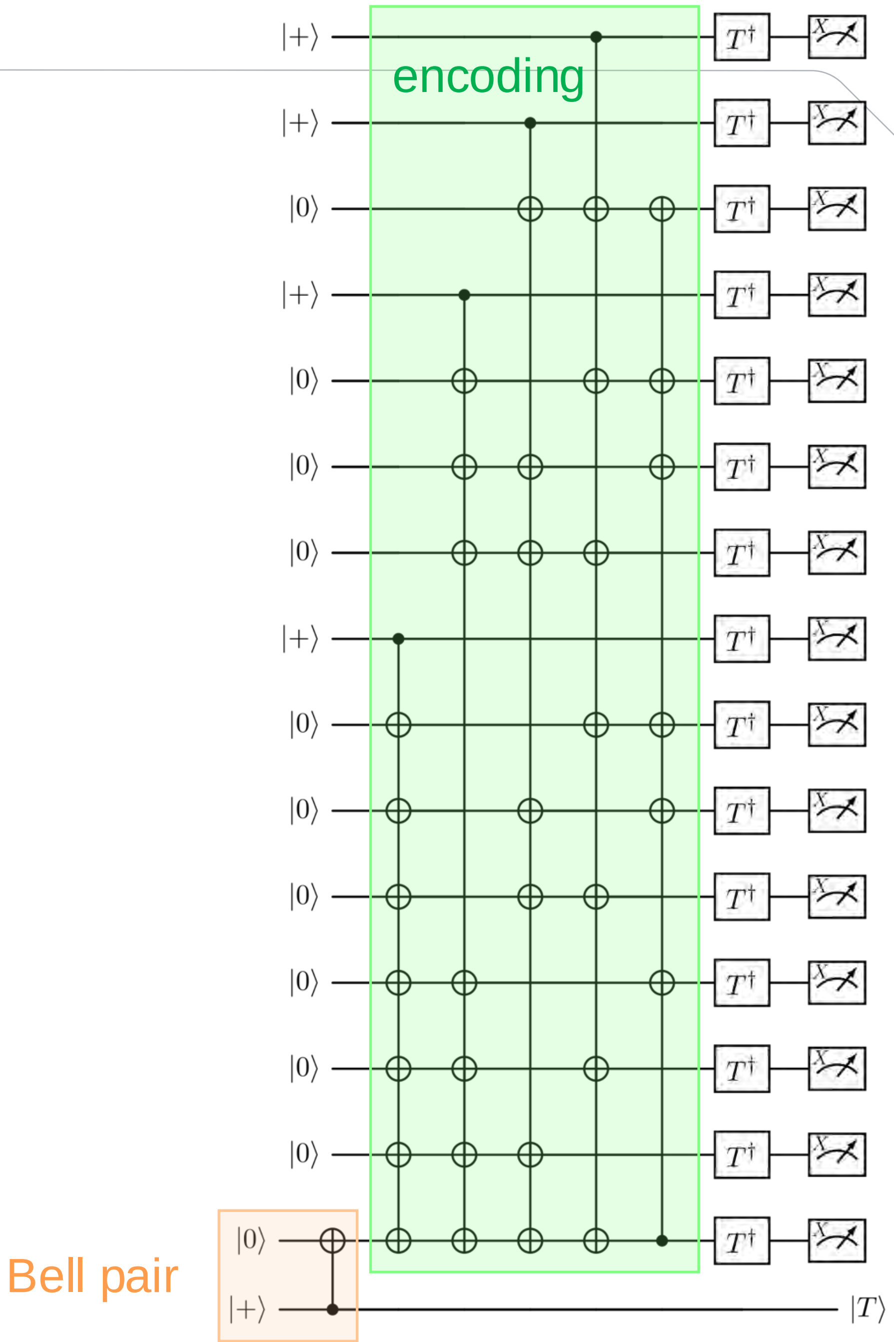
Logical qubits and operations at the logical level

Noise bias enables:

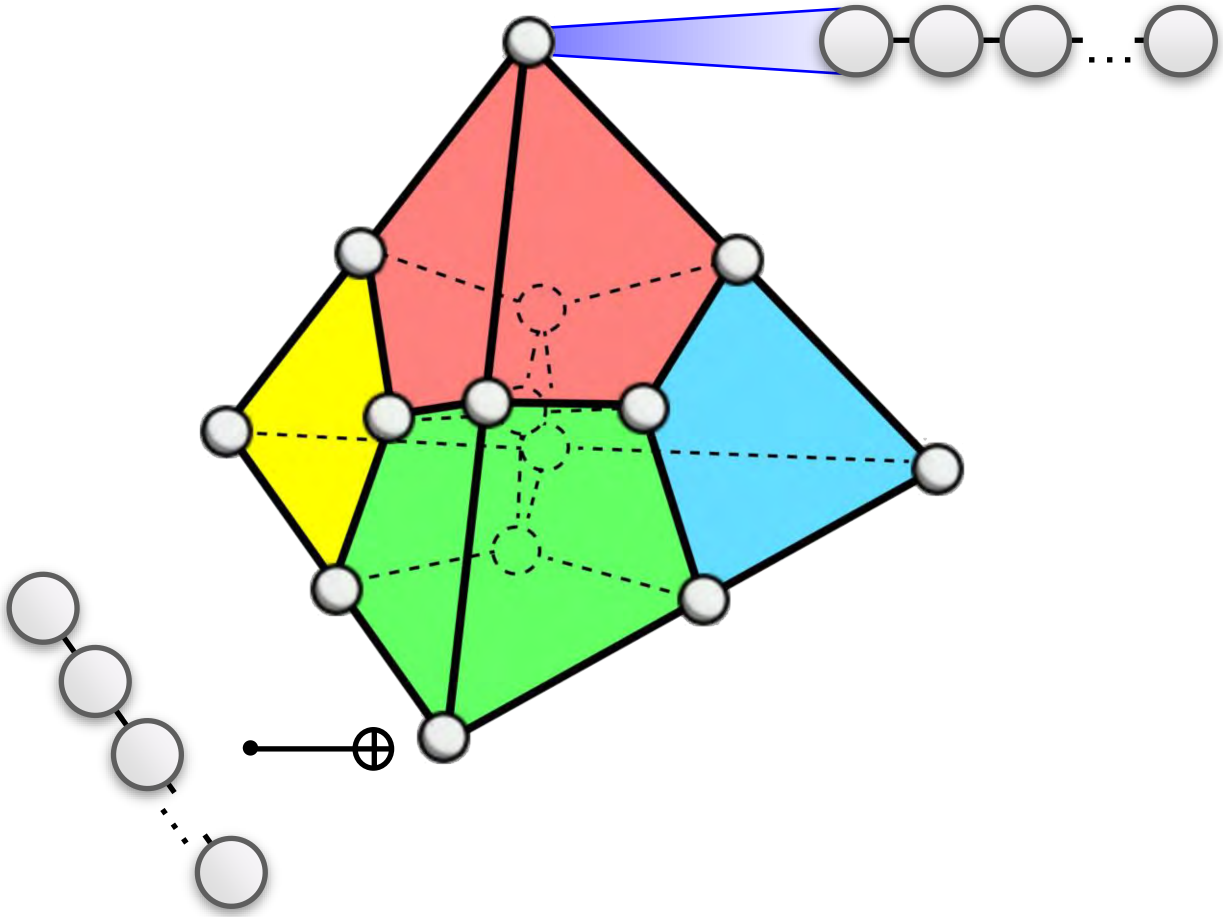
- To perform at distillation at physical level (using LDPC codes)

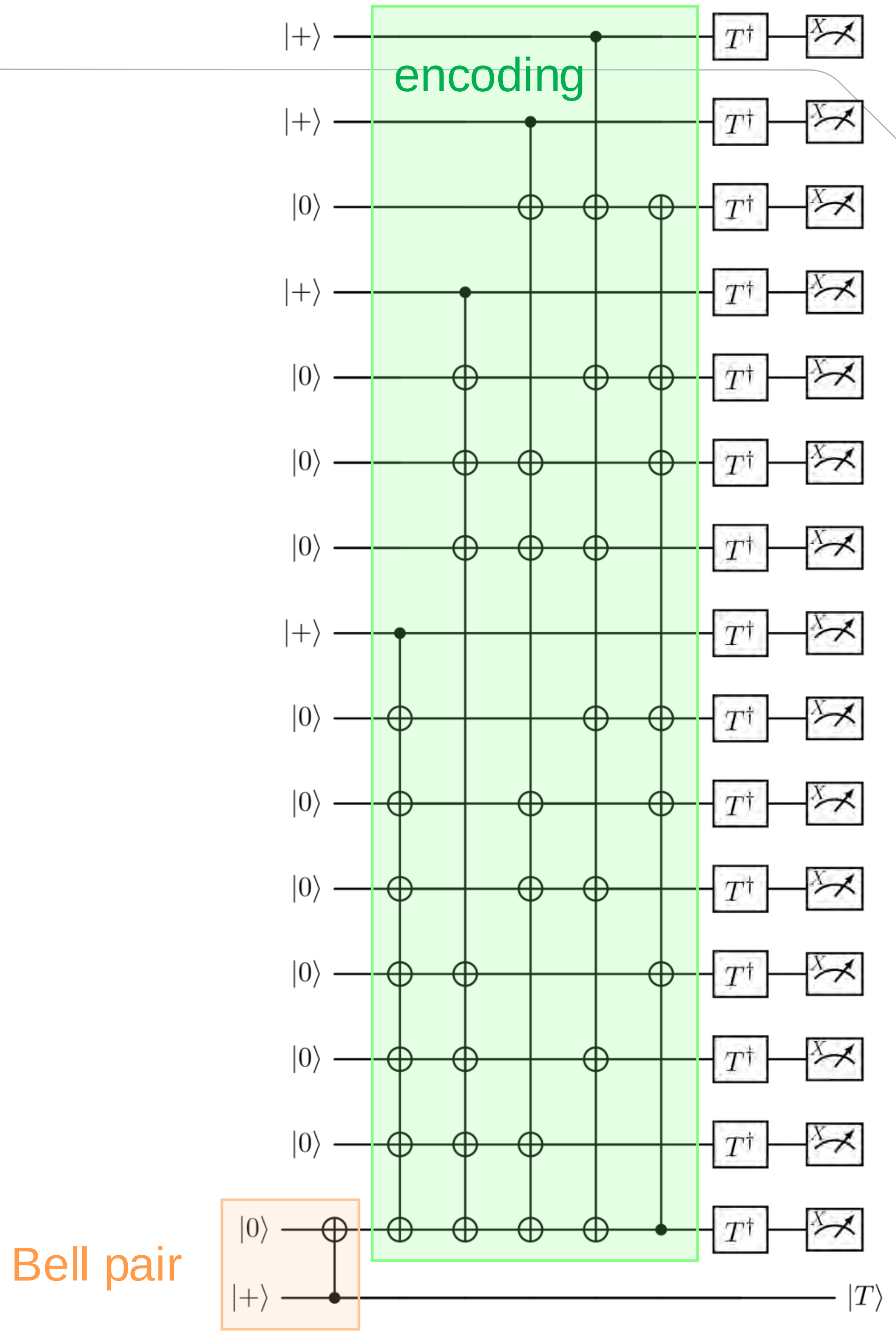
➢ Reduce the space cost
➢ Reduce the time cost



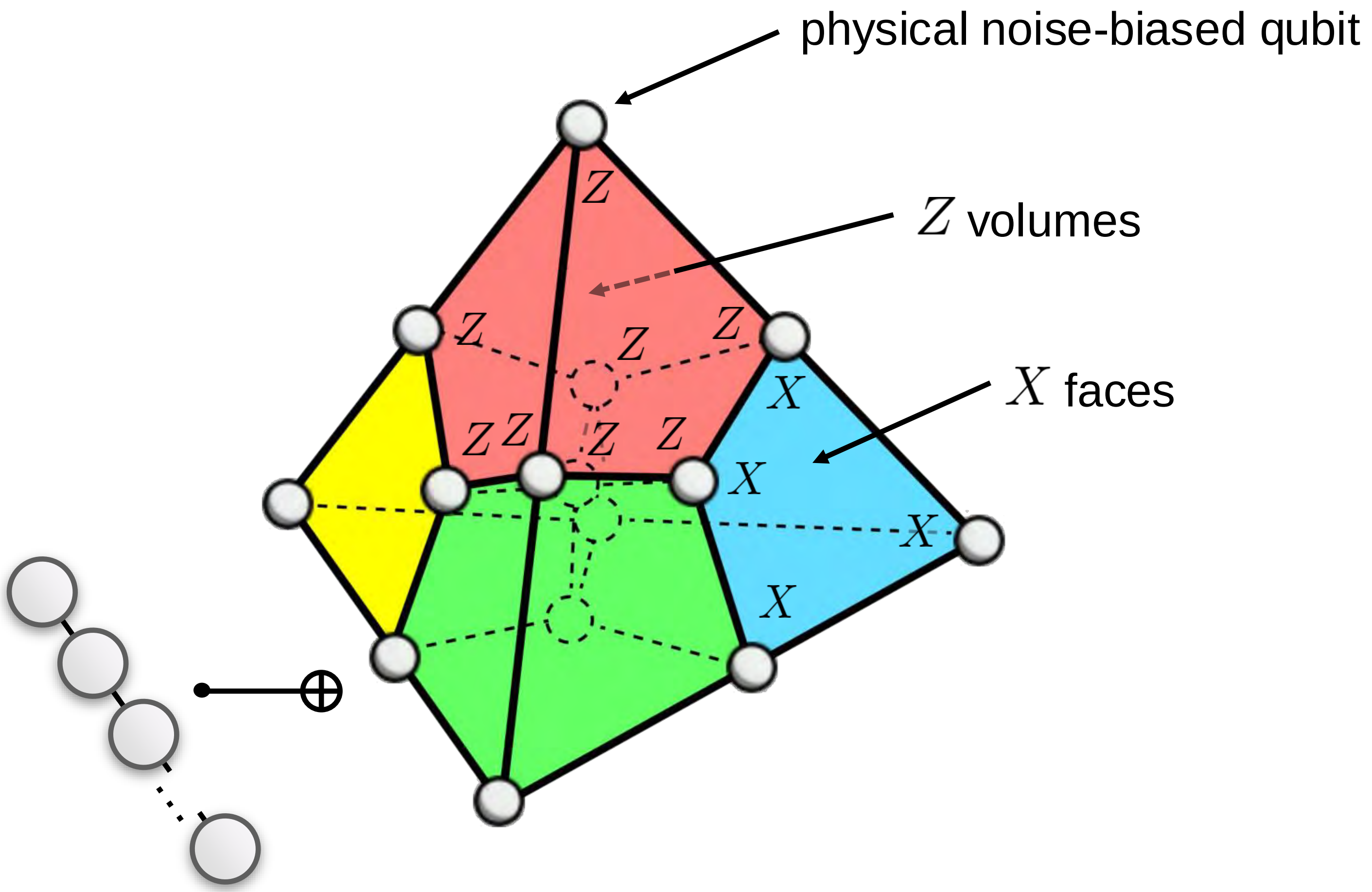

$|+\rangle$
$|0\rangle$
encoding
$T^\dagger$
X
Bell pair
$|T\rangle$

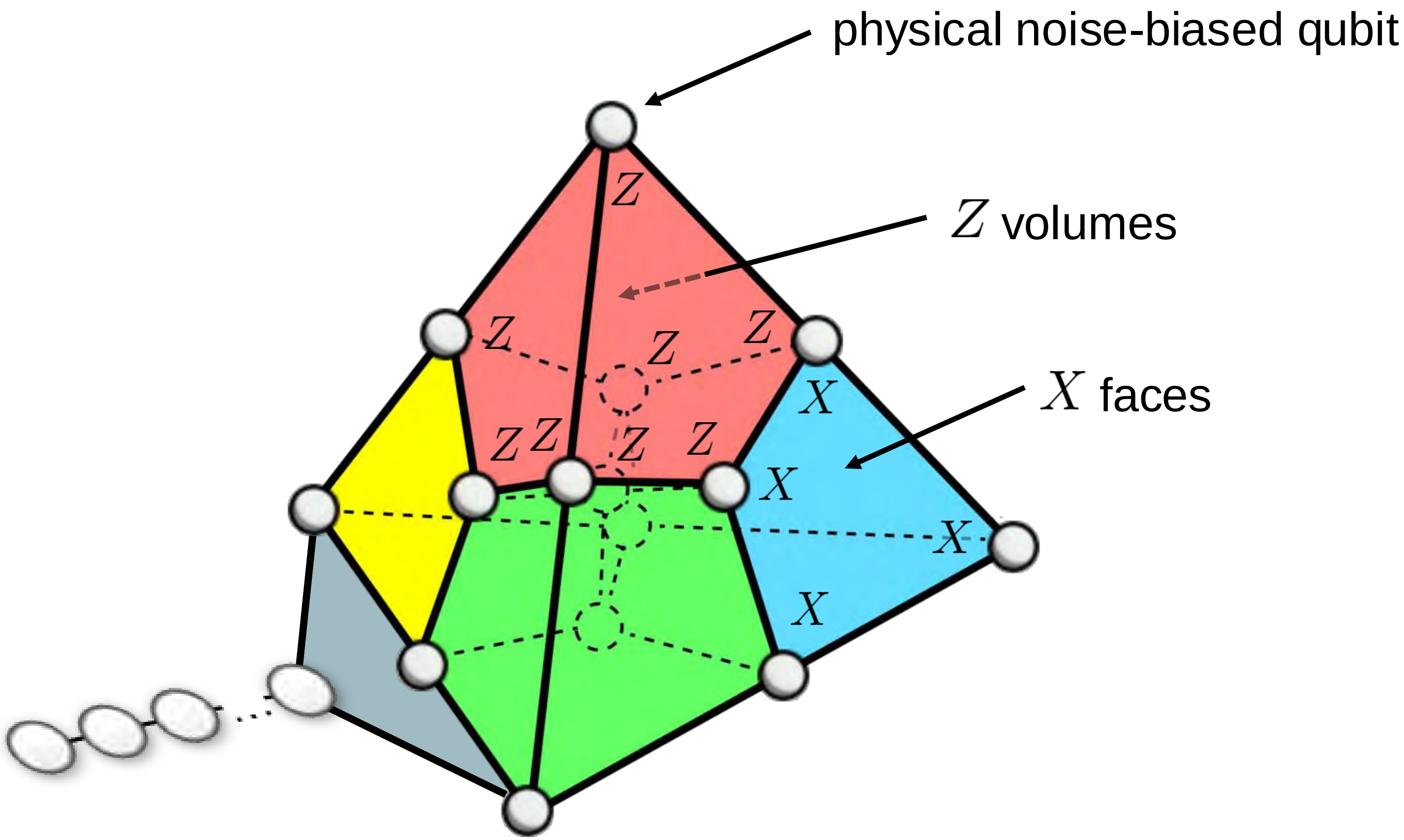
Distillation in 3D



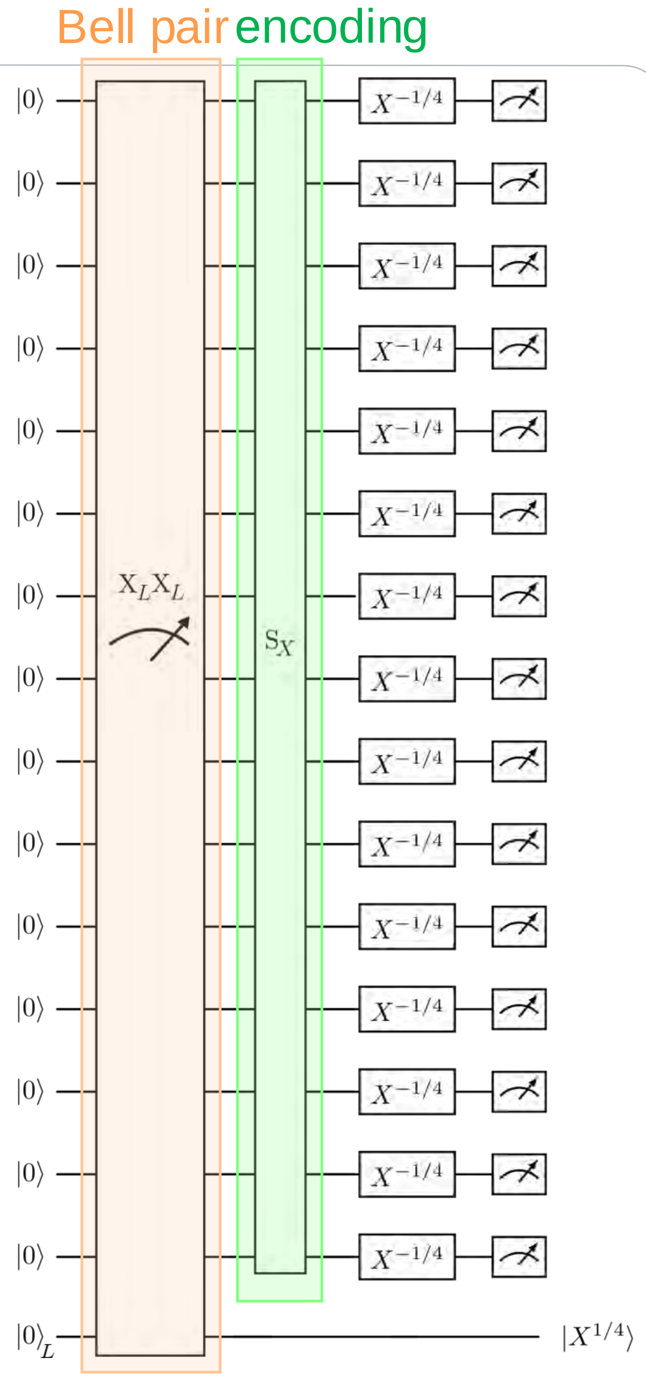


Bell pair





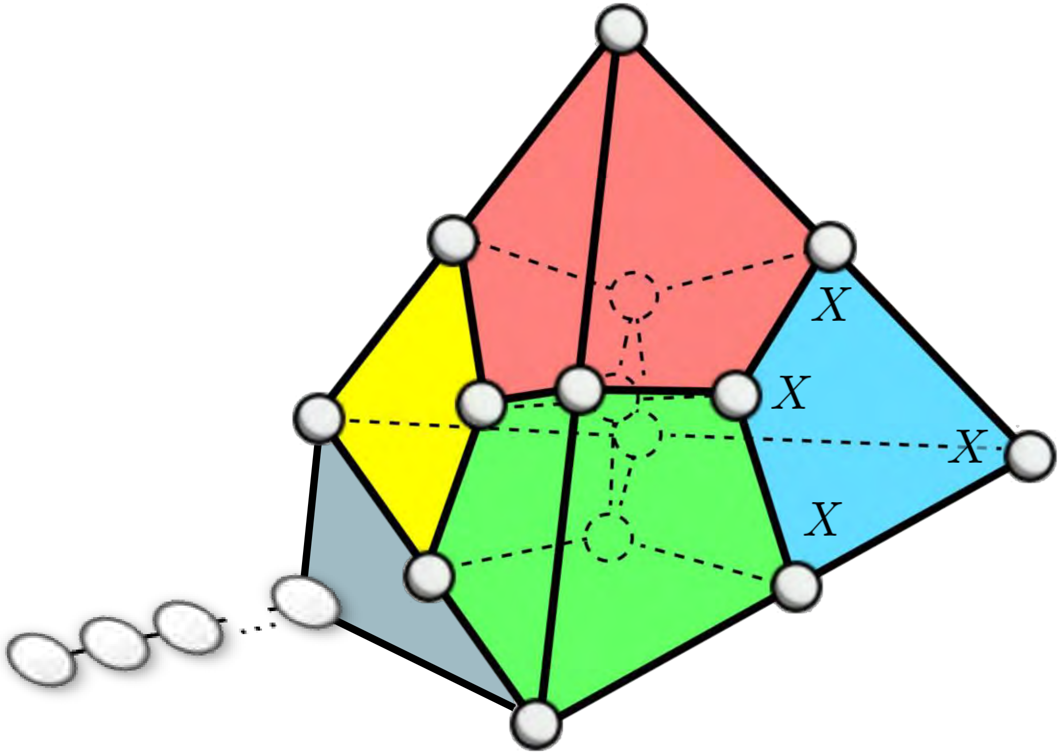
physical noise-biased qubit
Z volumes
X faces



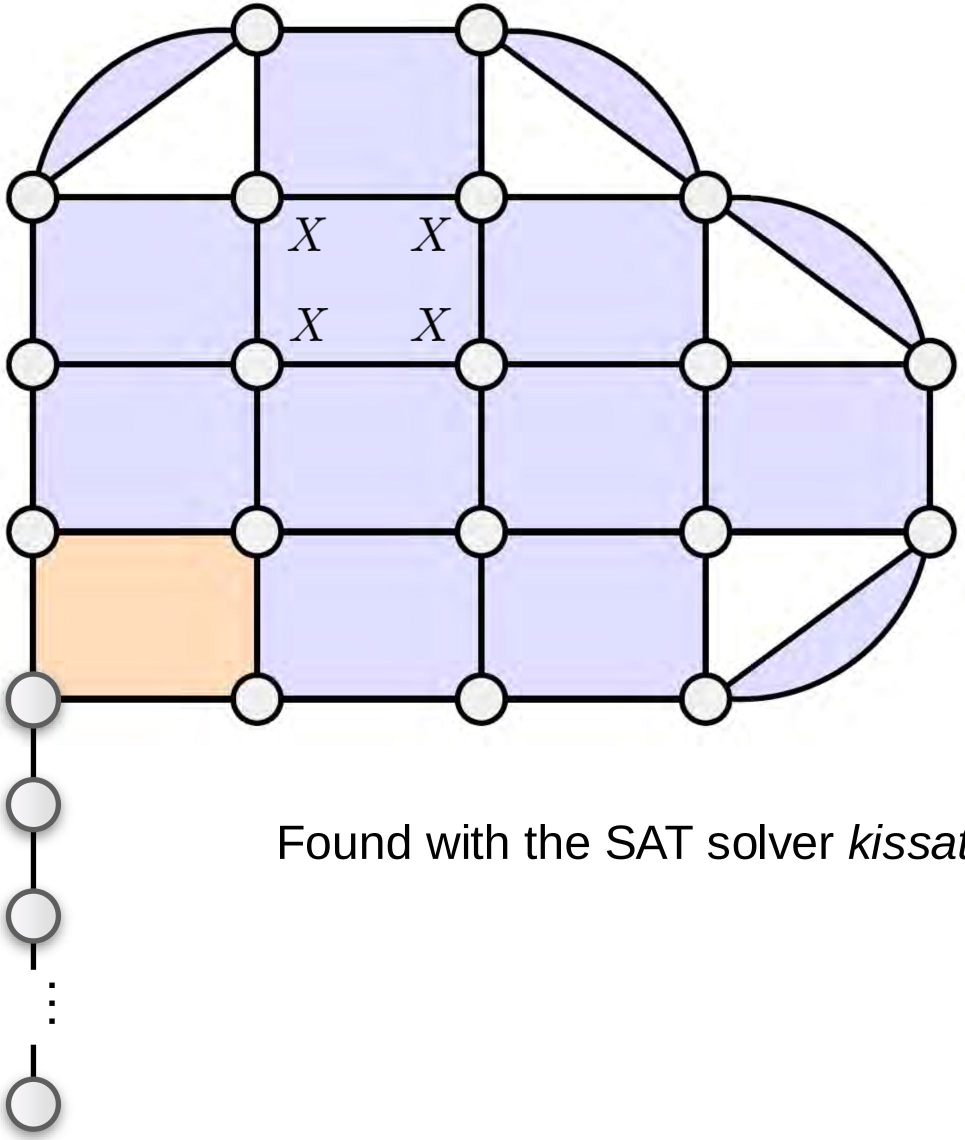
Bell pair
encoding
$|0\rangle$
$X_L X_L$
S_X
$X^{-1/4}$
$|0\rangle_L$
$|X^{1/4}\rangle$

Biased-noise qubit reduce the spatial dimension

➢ distillation at the physical level

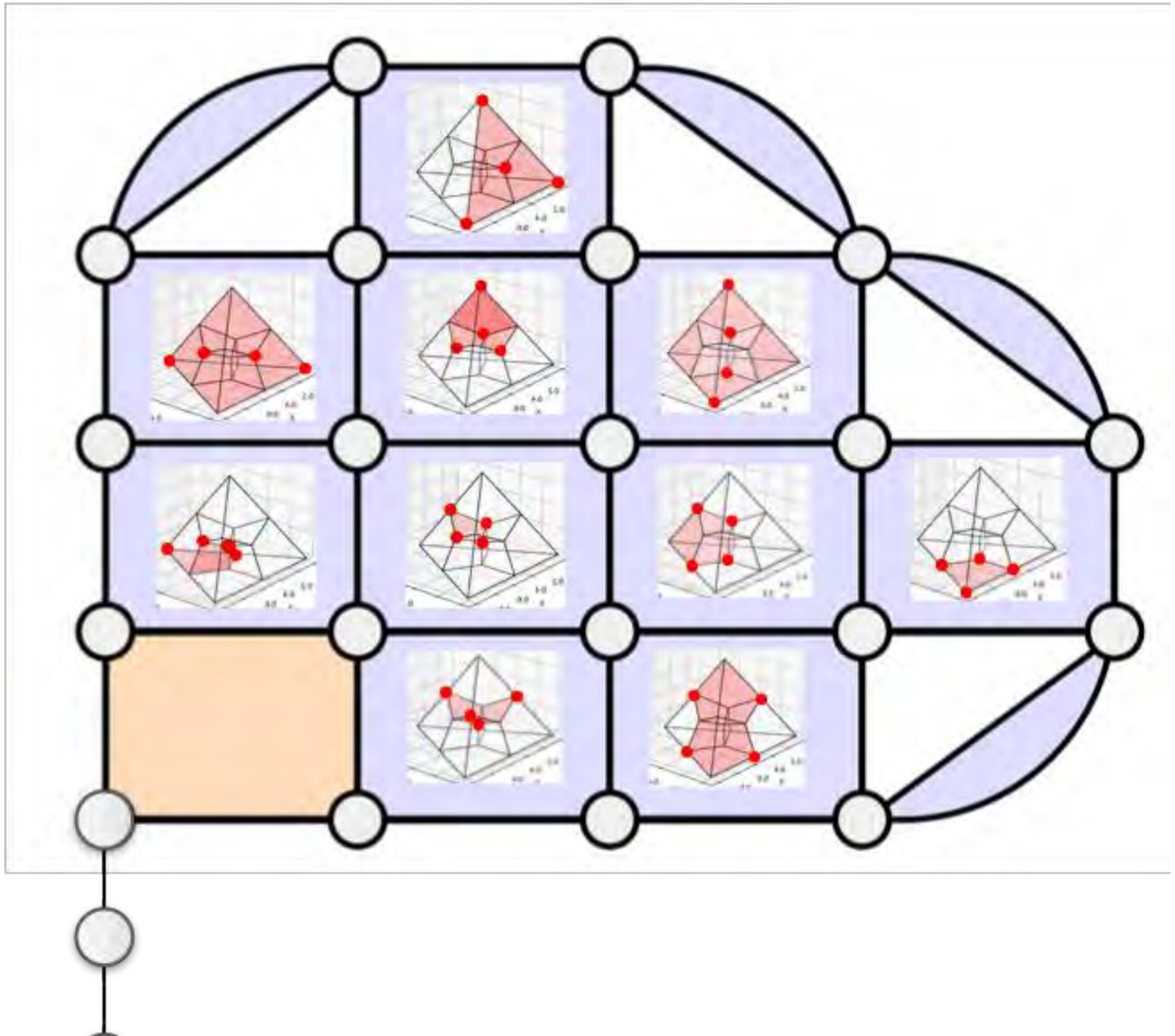


unfolding

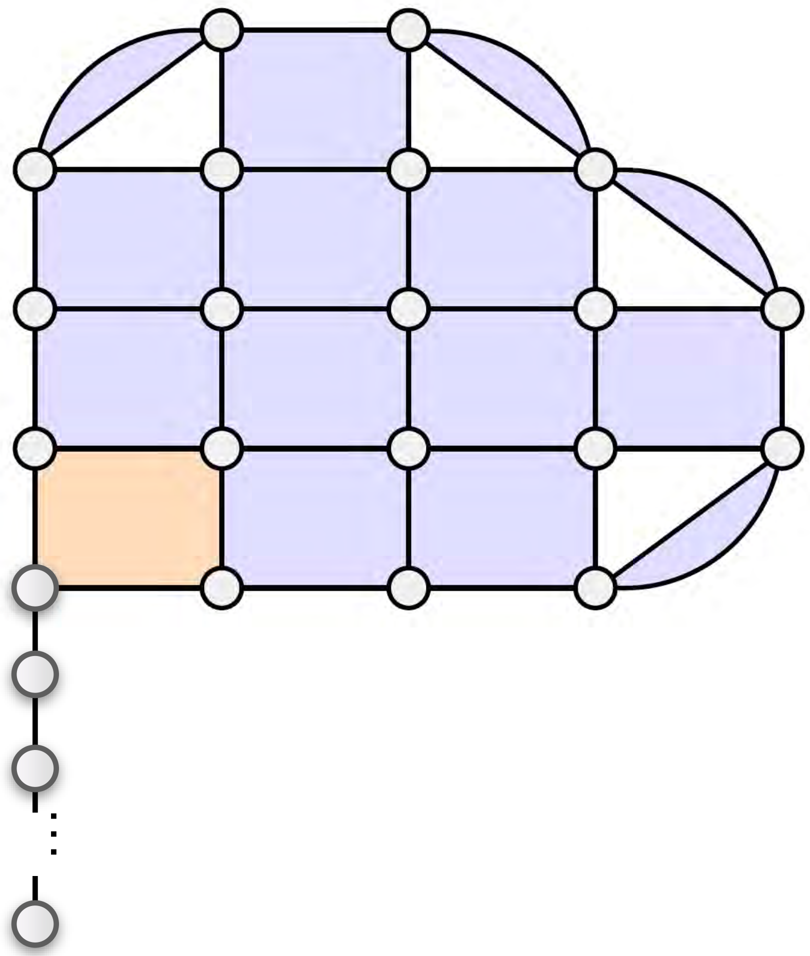


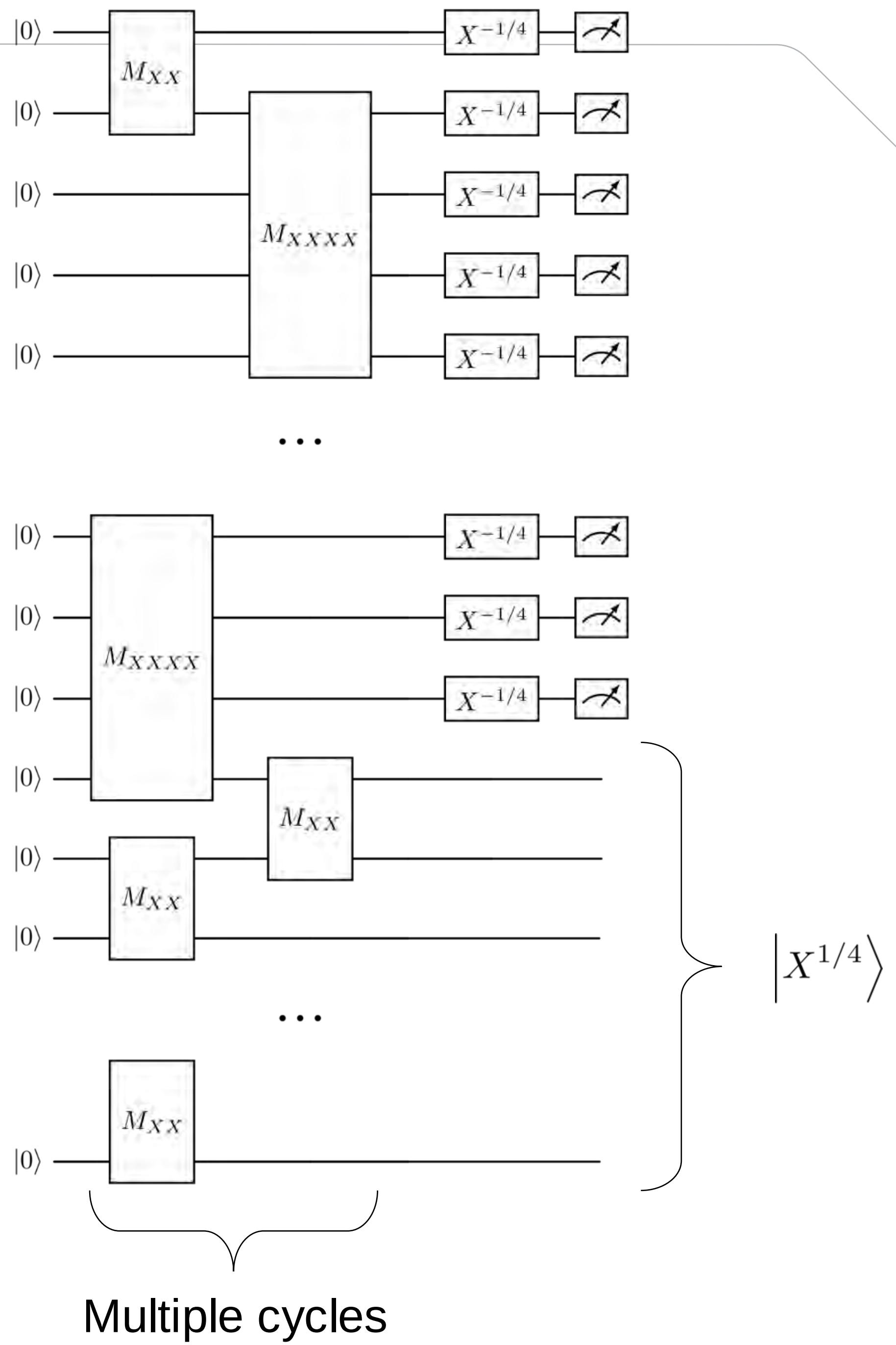
Found with the SAT solver *kissat*

Unfolding



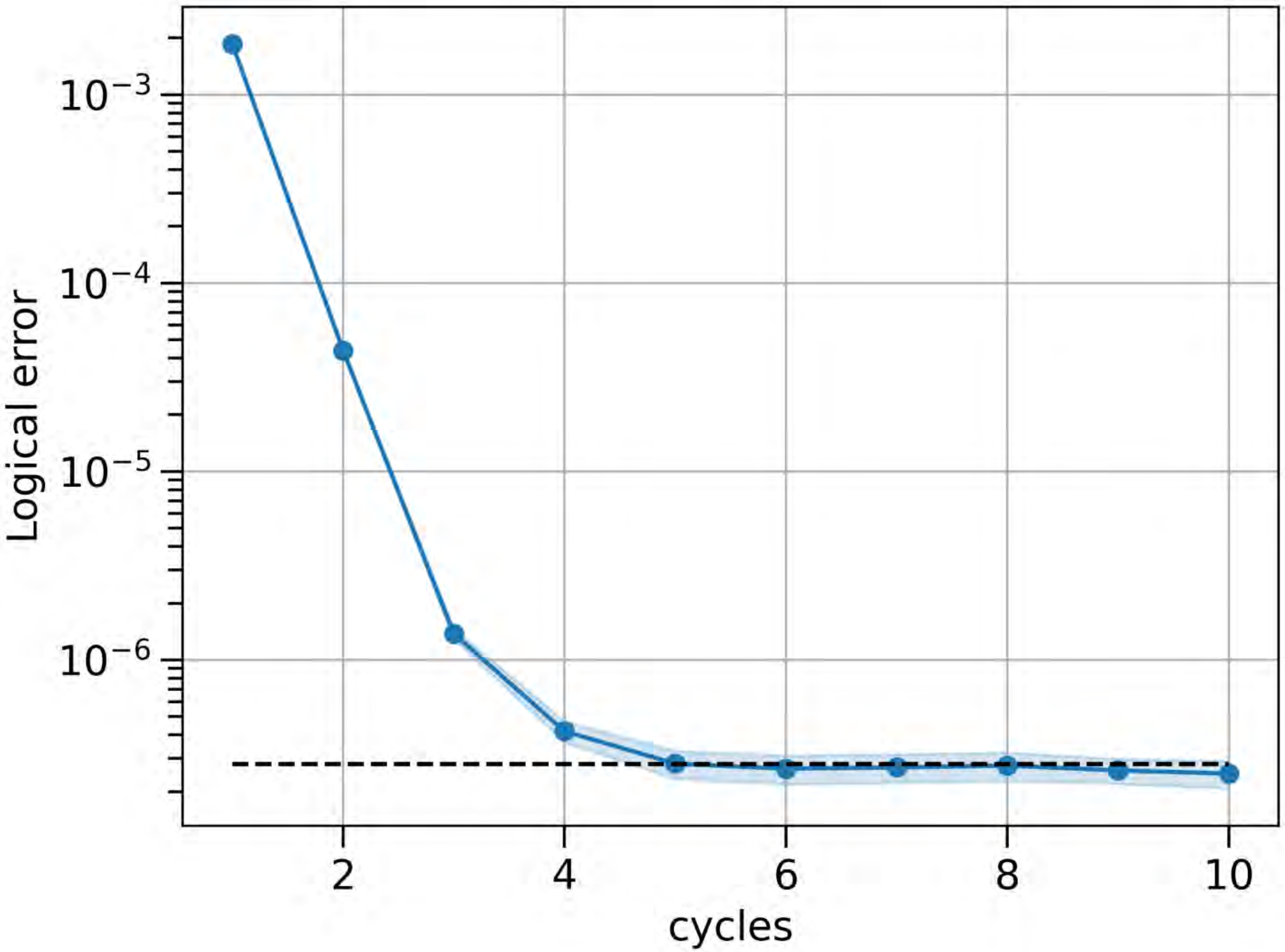
Distillation factory



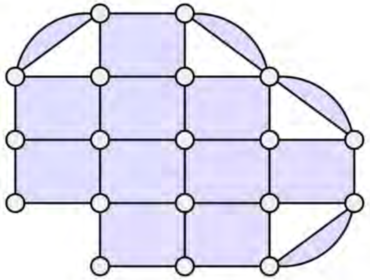


Numerics

Depolarizing noise $p_Z = 10^{-3}$



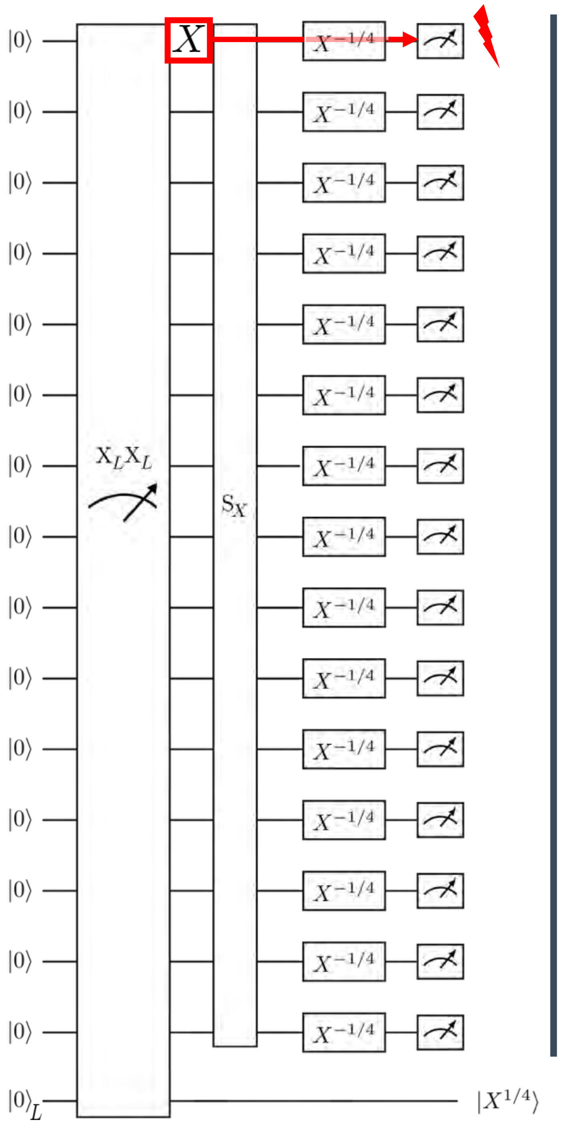
STIM + BPOSD



57 qubits
5.3 cycles (including post-selection)

$p_X = 0?$

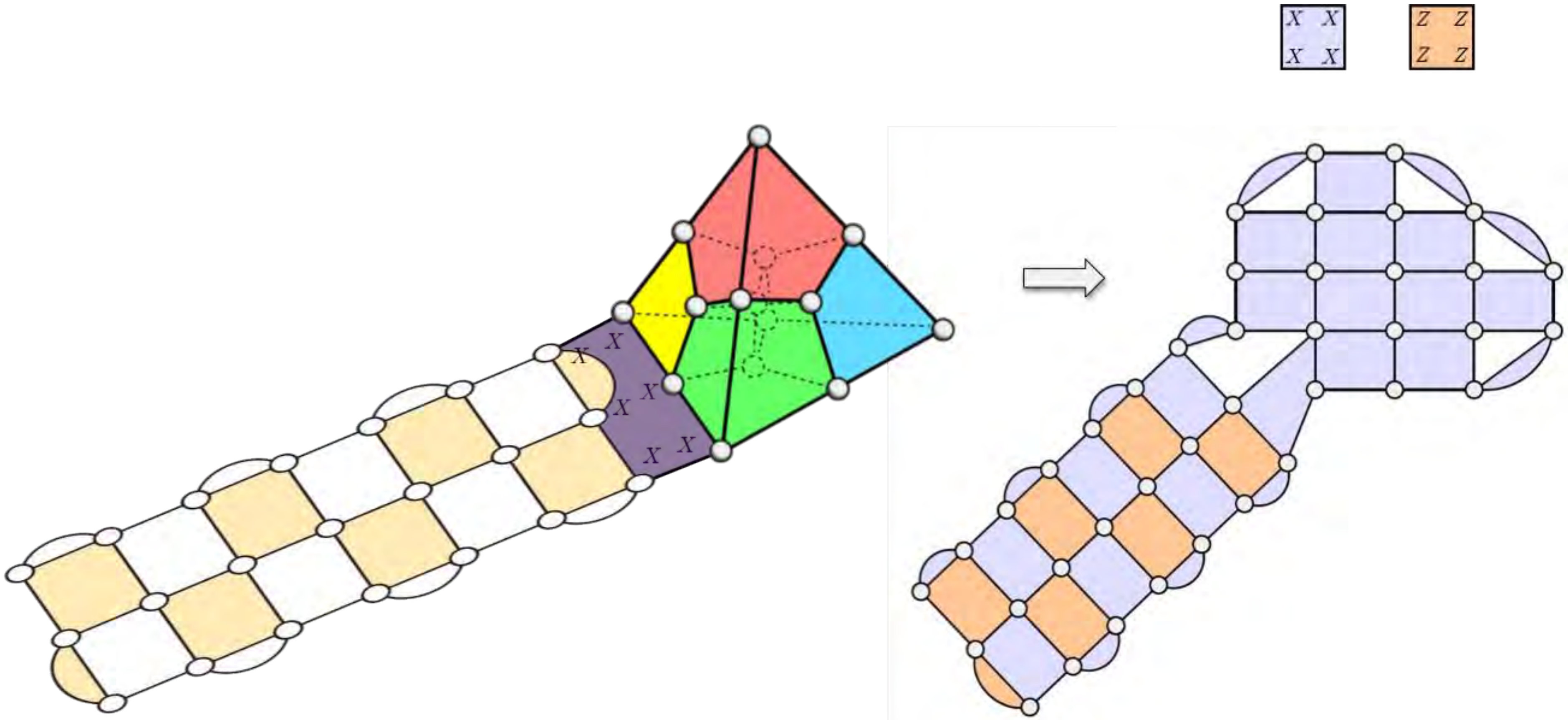
Bit-flip detection



Use the outer code to detect bitflip errors

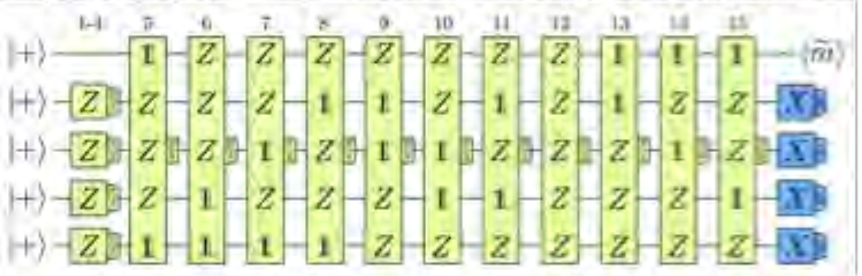
Detect up to 2 bitflips → $\eta \approx 30$

Merging in a bias tailored code



Comparison with 'unbiased' distillation

Magic State Distillation: Not as Costly as You Think

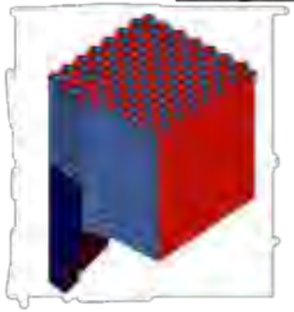


Litinski, Quantum 3, 205, 2019

4,620 qubits
42.6 rounds
197,000 Qubitcycles

$\Rightarrow$ $10^{-3} \rightarrow 4.5 \times 10^{-8}$

Magic state cultivation [...]

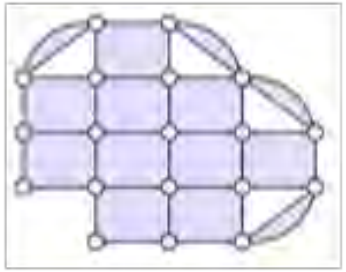


Gidney *et al*, arXiv:2409, 2024

500 qubits
50 rounds
25,000 Qubitcycles ($/10$)

$\Rightarrow$ $10^{-3} \rightarrow 10^{-6}$

Biased noise distillation

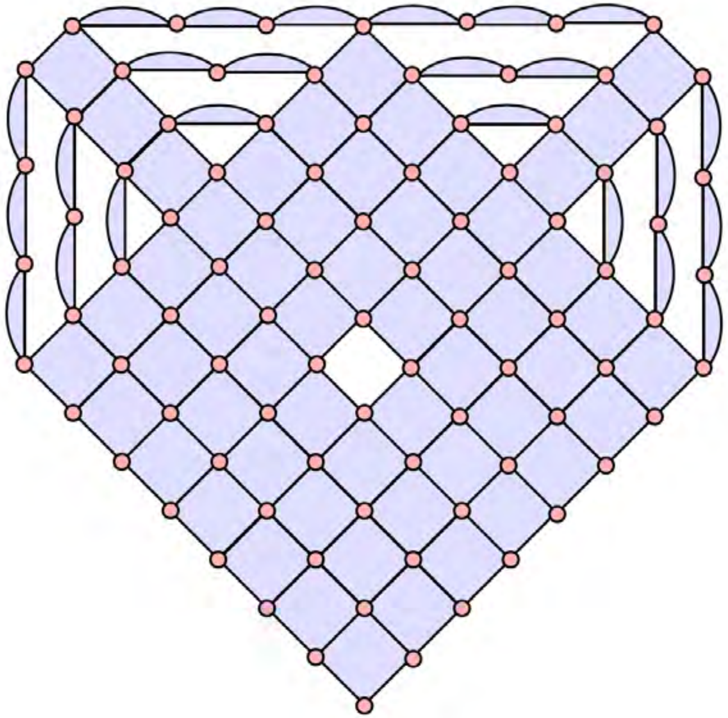


$\eta = 5 \times 10^5$
57 qubits
5.3 rounds
300 Qubitcycles ($/650$)

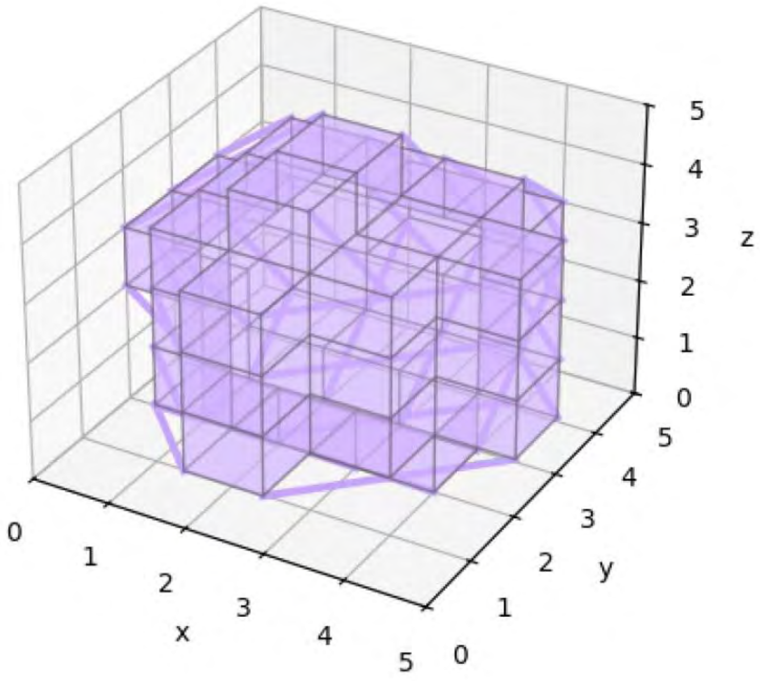
$\Rightarrow$ $10^{-3} \rightarrow 10^{-7}$ $\Leftarrow$

$\eta \approx 30$
179 qubits
9 rounds
$\approx$ 1,600 Qubitcycles ($/120$)

Higher distance unfolding



distance 4 in 2D: $10^{-3} \rightarrow 8 \times 10^{-9}$



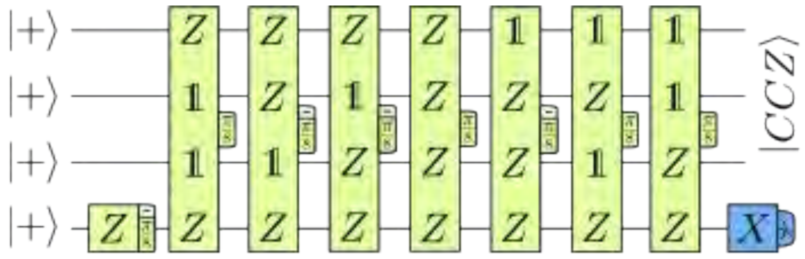
distance 7 in 2D: $10^{-3} \rightarrow 10^{-17}$

Vivien Londe, arxiv:2605.06284

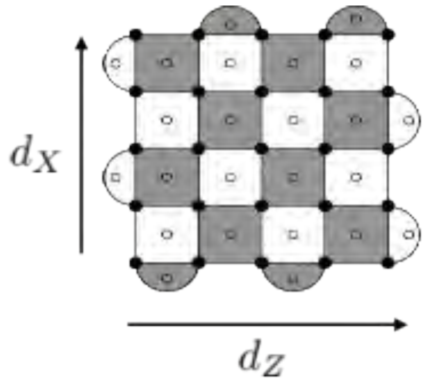
03

SIMULTANEOUS LATTICE SURGERY

2^{nd} layer of distillation: a compilation problem

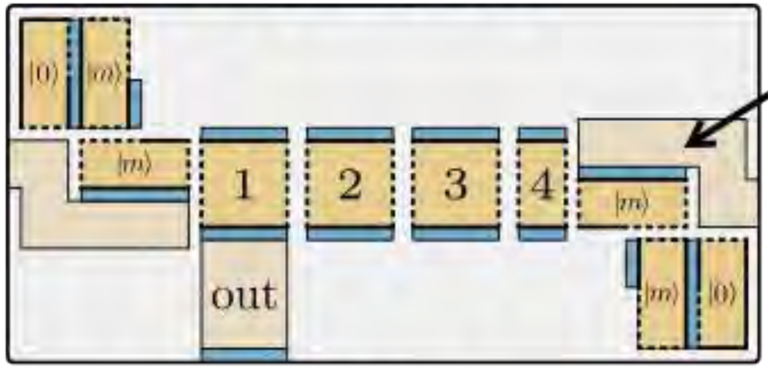


$$p_{\text{out},1} \longrightarrow p_{\text{out},2} \geq 28(p_{\text{out},1})^2$$

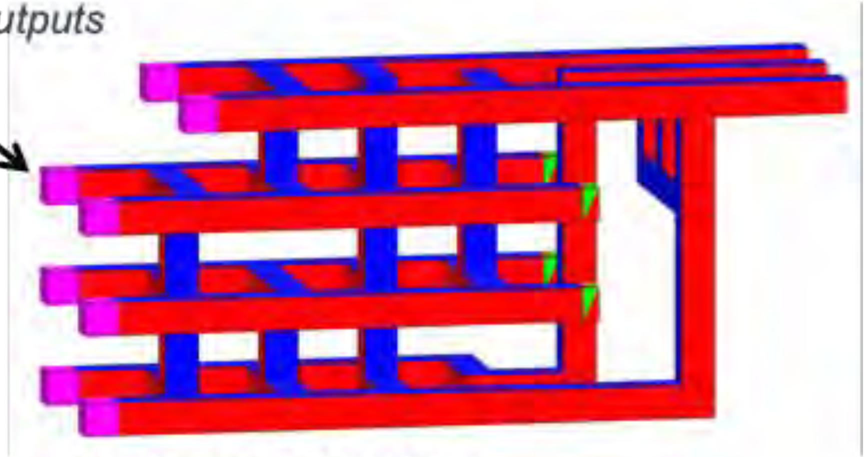


Surface code architectures :

First level factories outputs

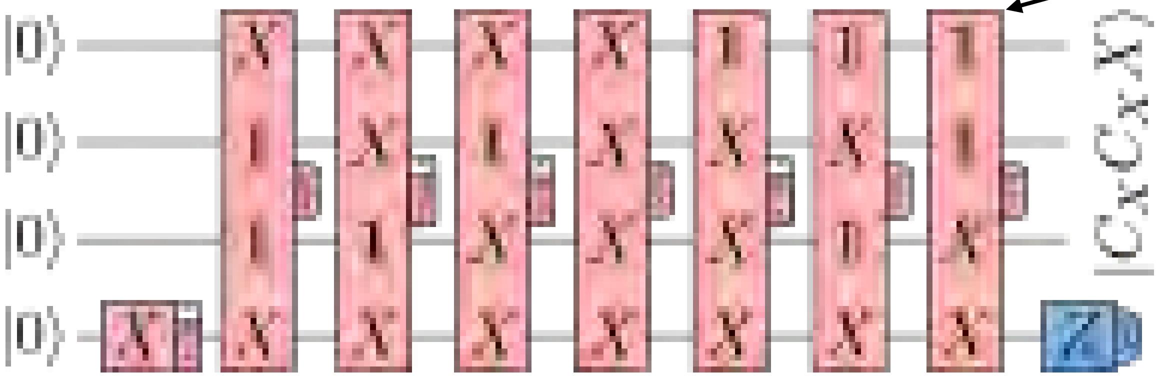


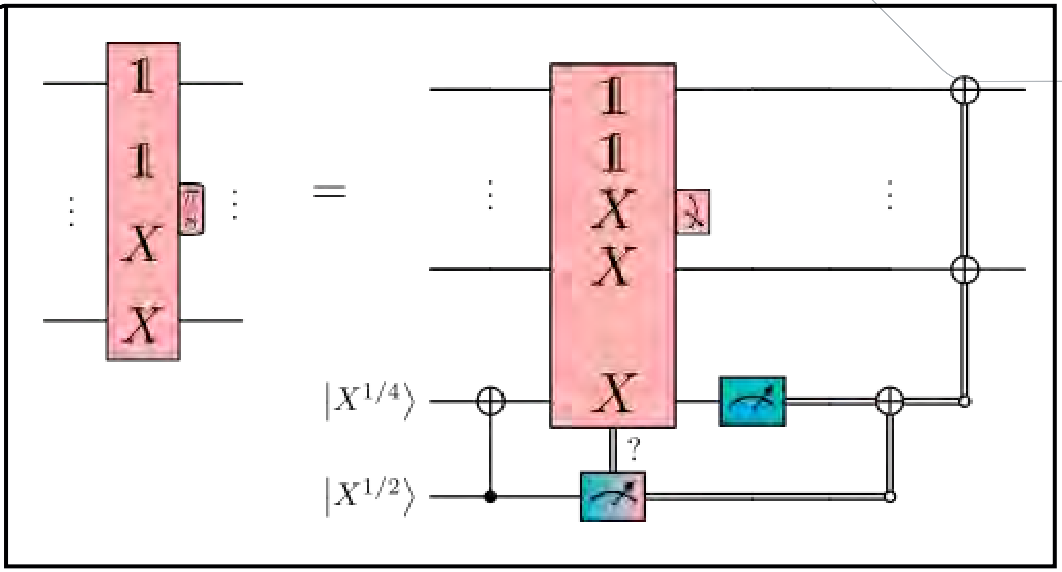
Litinski, Quantum 3, 205, 2019



Gidney, arxiv:2409.17595

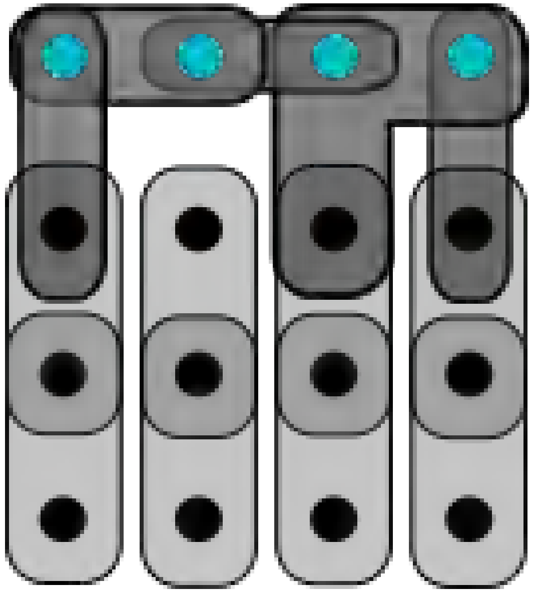
Multi qubit X rotations





Lattice surgery for phase-flip repetition code:

- data qubits
- Rep. Code X stabilizers
- routing qubits Initialized in $|0\rangle$
- Merging 2 qubits X stabilizers
- Merging 3 qubits X stabilizer



$$\bigoplus_{m \in \text{dark grey stabs}} M_m$$

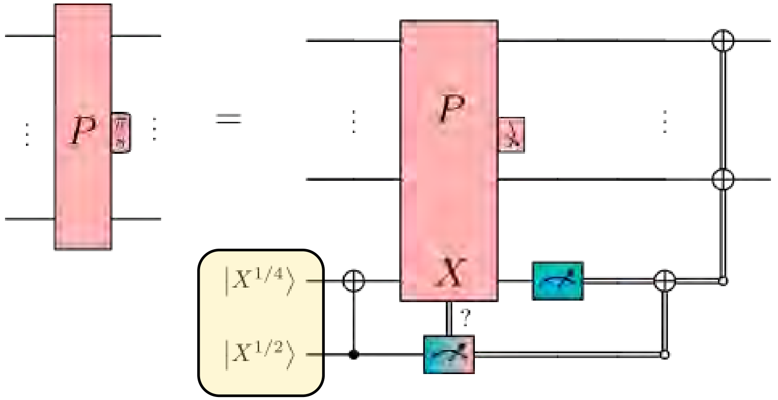
$$\Downarrow$$

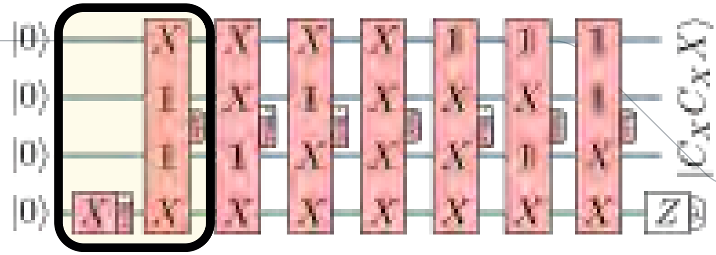
$$M_{X_1 X_3 X_4}$$

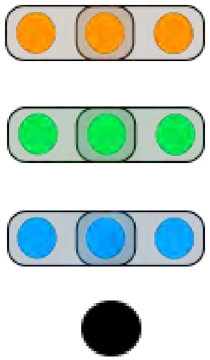
Timelike distance : d_m

Gouzien *et al*, Physical Review Letters 131, 2023

Multi qubit X rotations





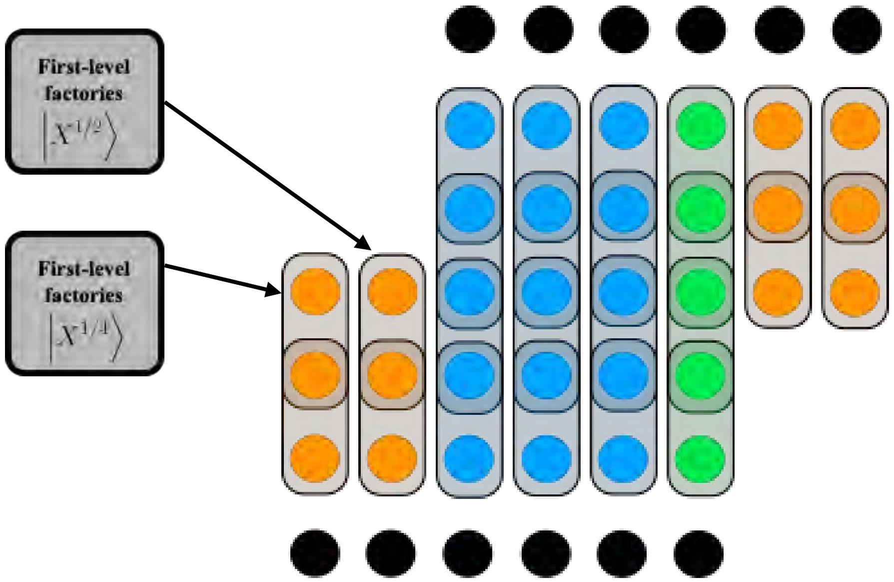


Level 1
magic states

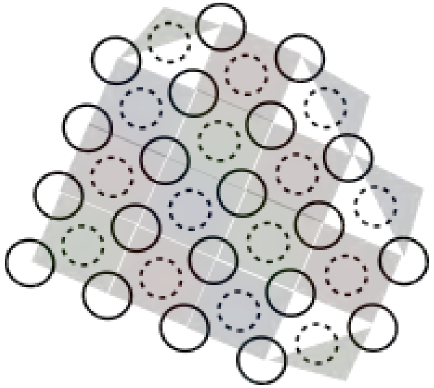
Level 2
Check qubit

Level 2
magic states

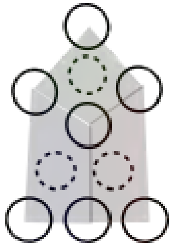
Routing qubits



Unfolded distillation :



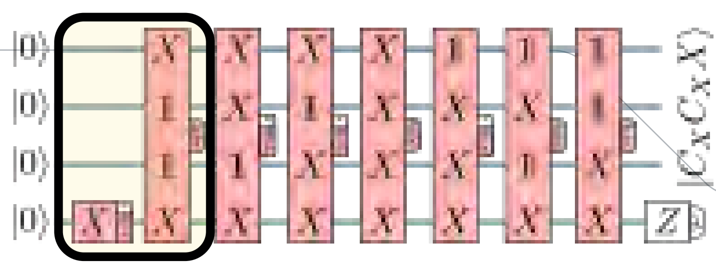
5.5 cycles
35 cat qubits
$p_{\mathrm{in},Z} = 1 \times 10^{-3}$
$p_{\mathrm{out}}^{(1)} = 3 \times 10^{-7}$

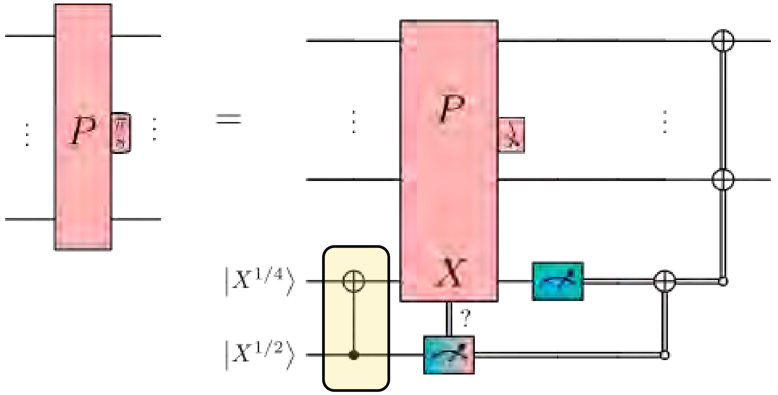


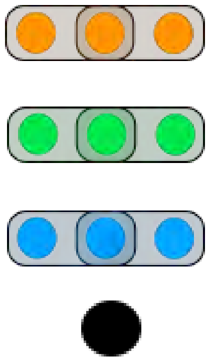
5 cycles
10 cat qubits
$p_{\mathrm{in},Z} = 1 \times 10^{-3}$
$p_{\mathrm{out}}^{(1)} = 6 \times 10^{-8}$

Ruiz *et al*, arXiv:2507.12511, 2025

Multi qubit X rotations





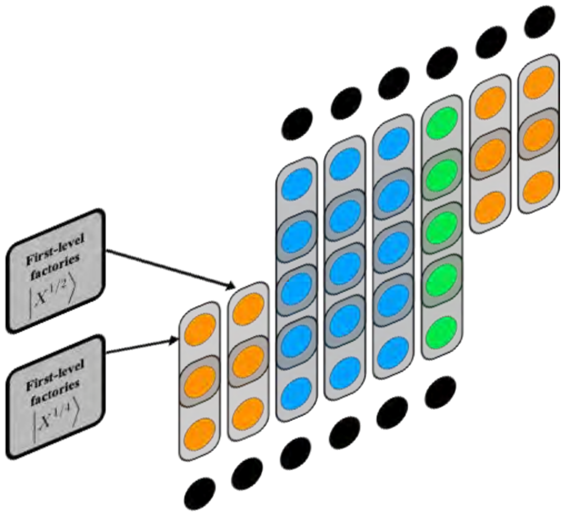


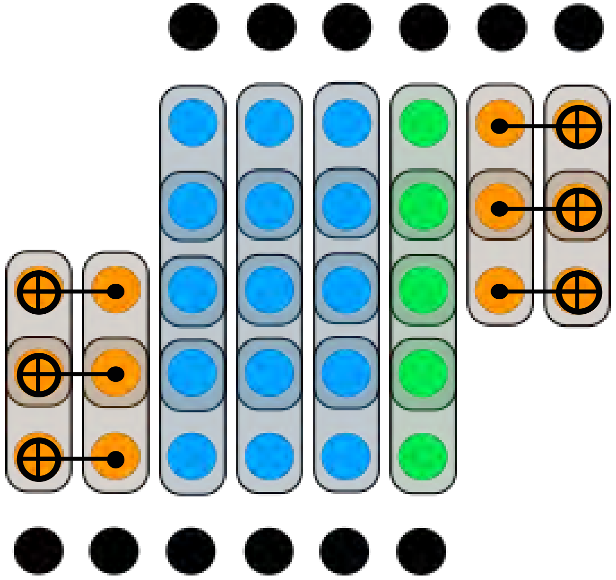
Level 1
magic states

Level 2
Check qubit

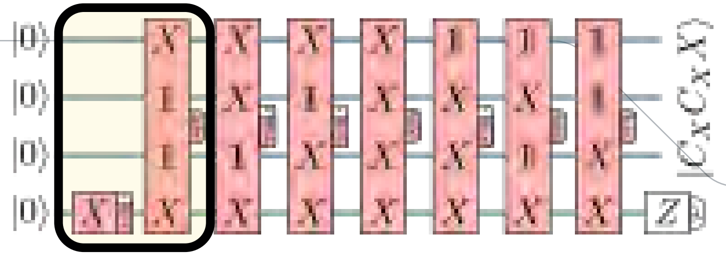
Level 2
magic states

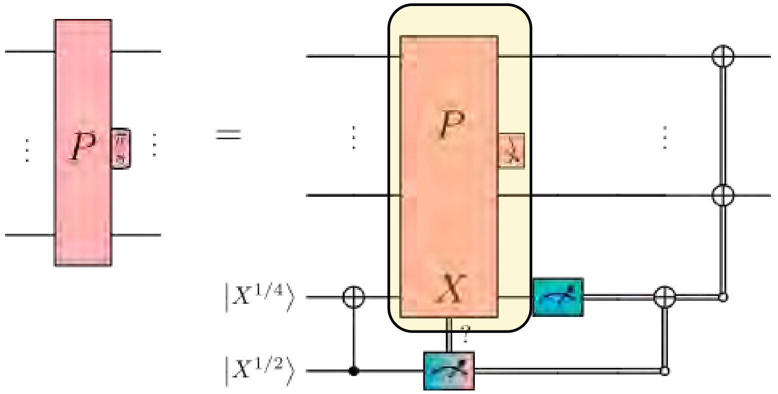
Routing qubits

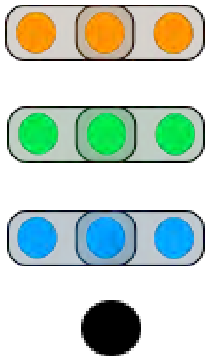




Multi qubit X rotations





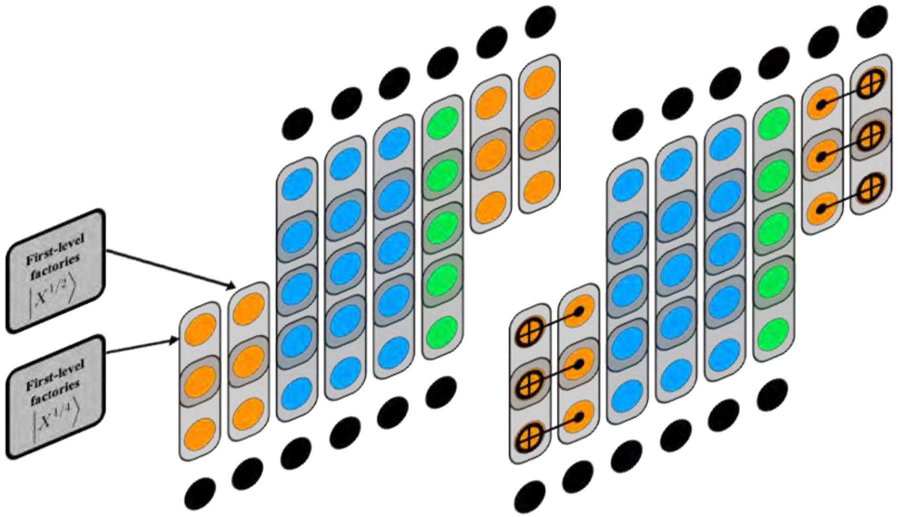


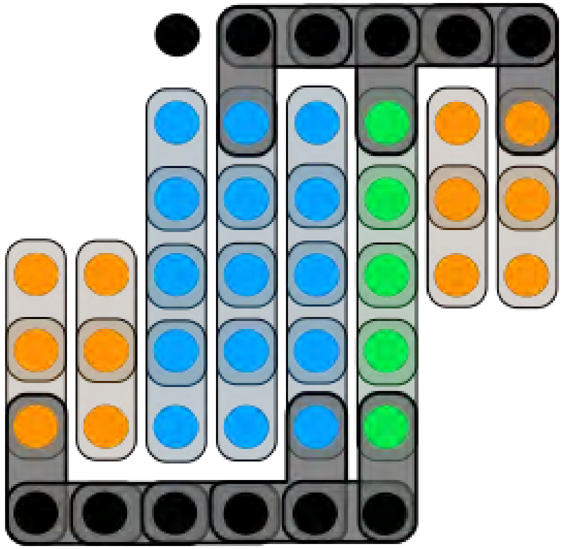
Level 1 magic states

Level 2 Check qubit

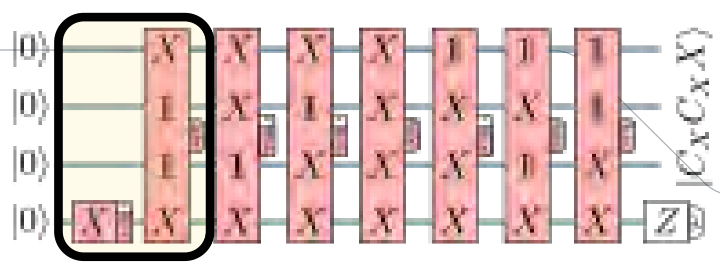
Level 2 magic states

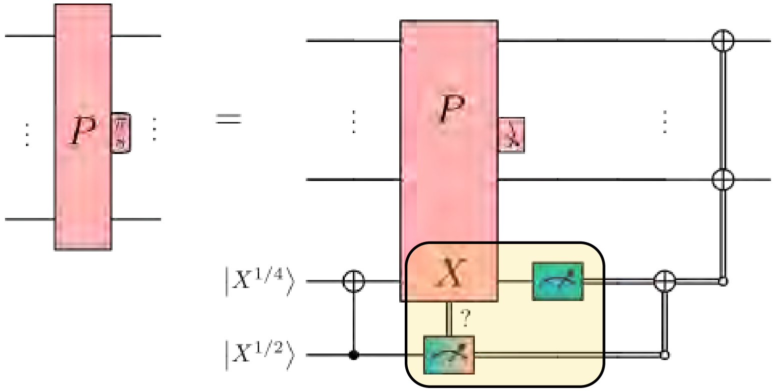
Routing qubits

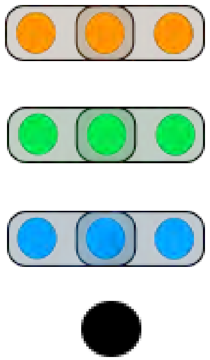




Multi qubit X rotations





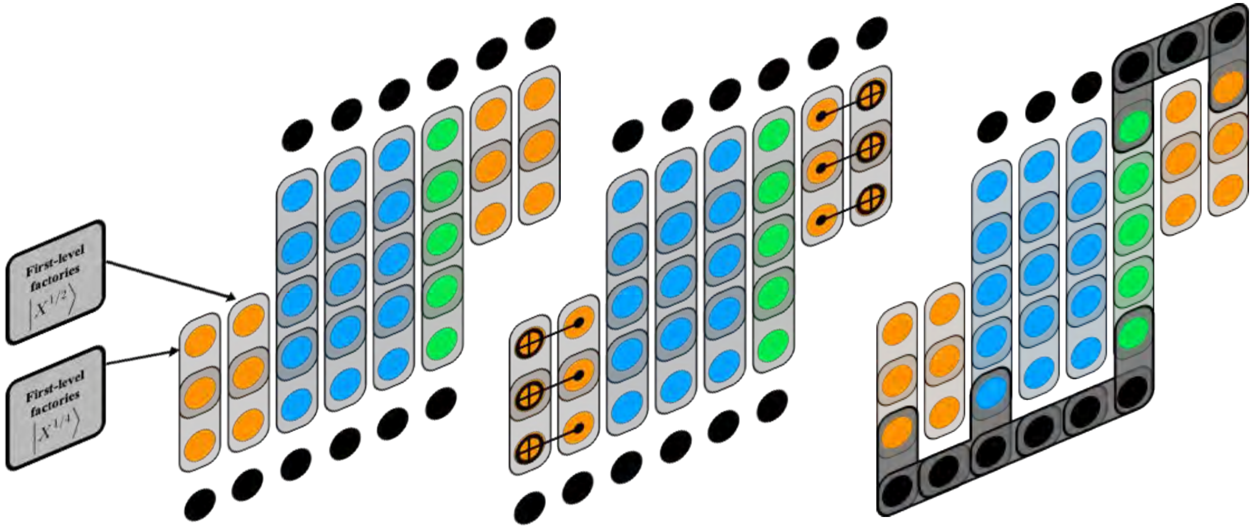


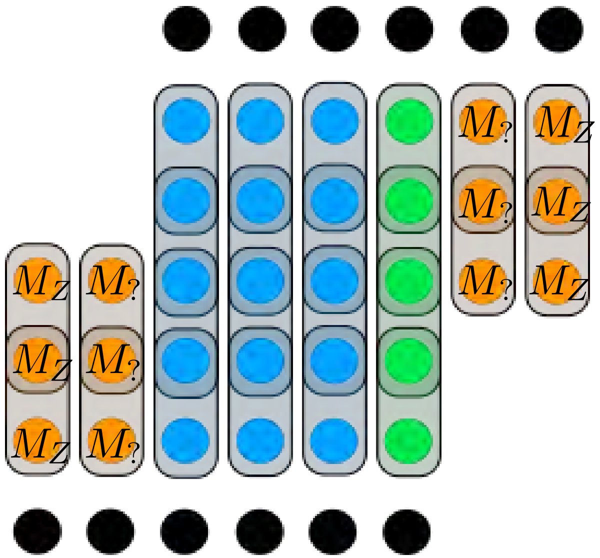
Level 1
magic states

Level 2
Check qubit

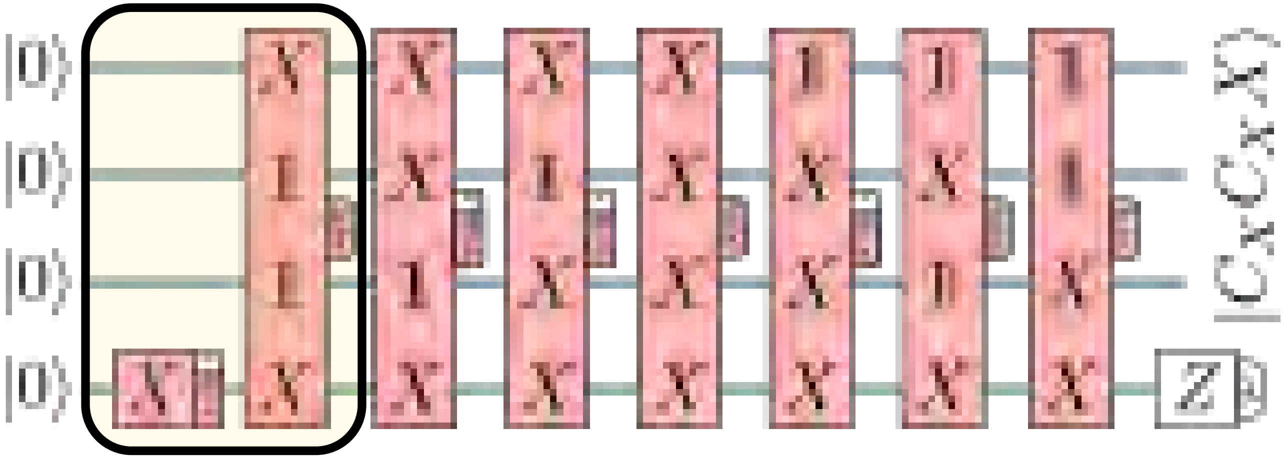
Level 2
magic states

Routing qubits





Multi qubit X rotations



Level 1
magic states

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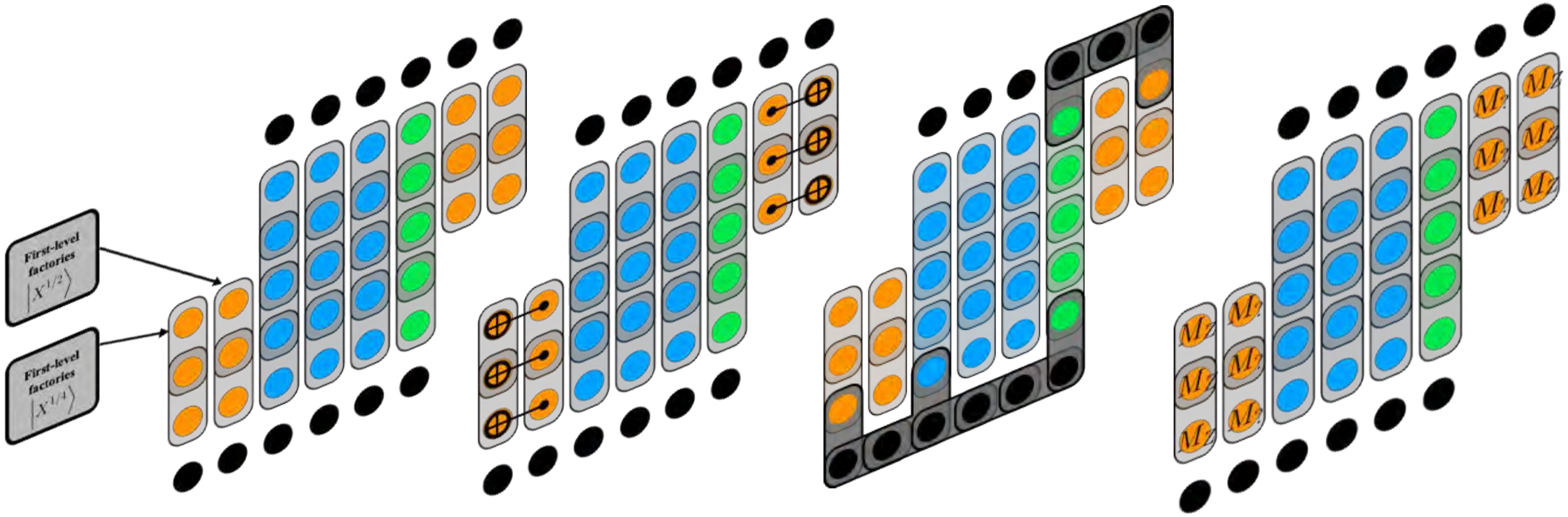
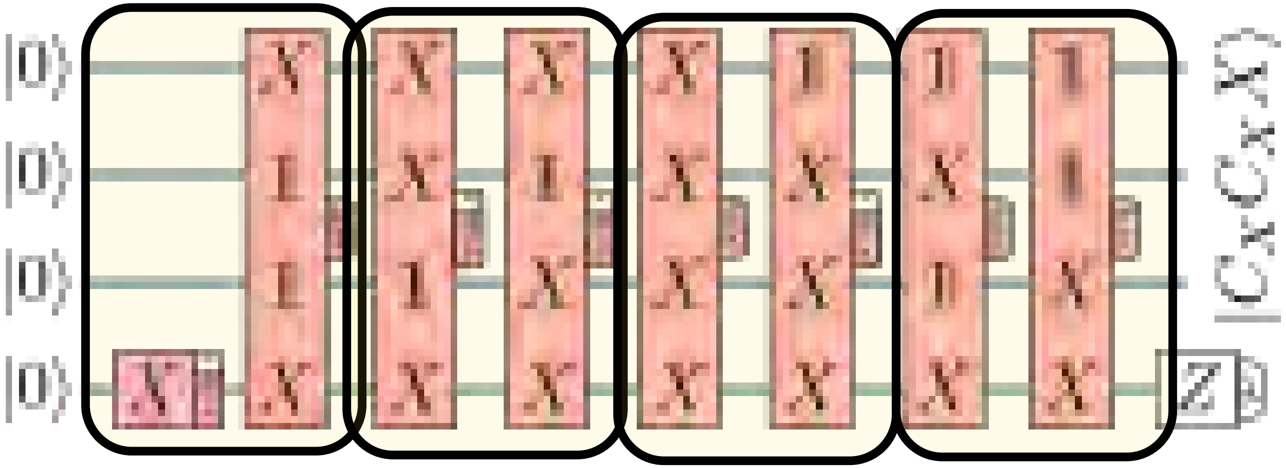
Check qubit

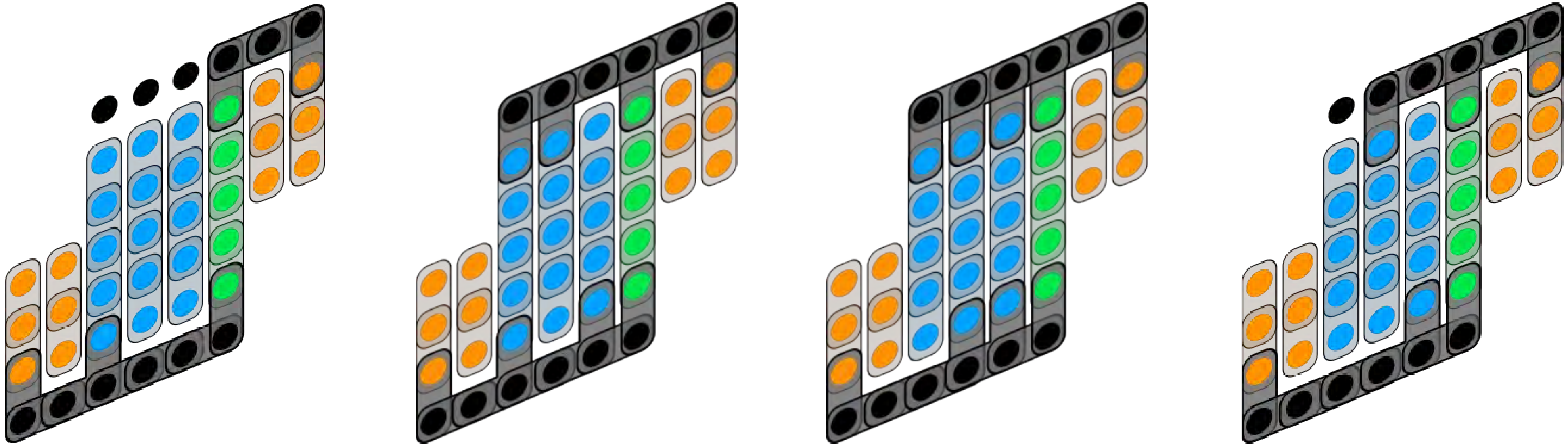
Level 2
magic states

Routing qubits



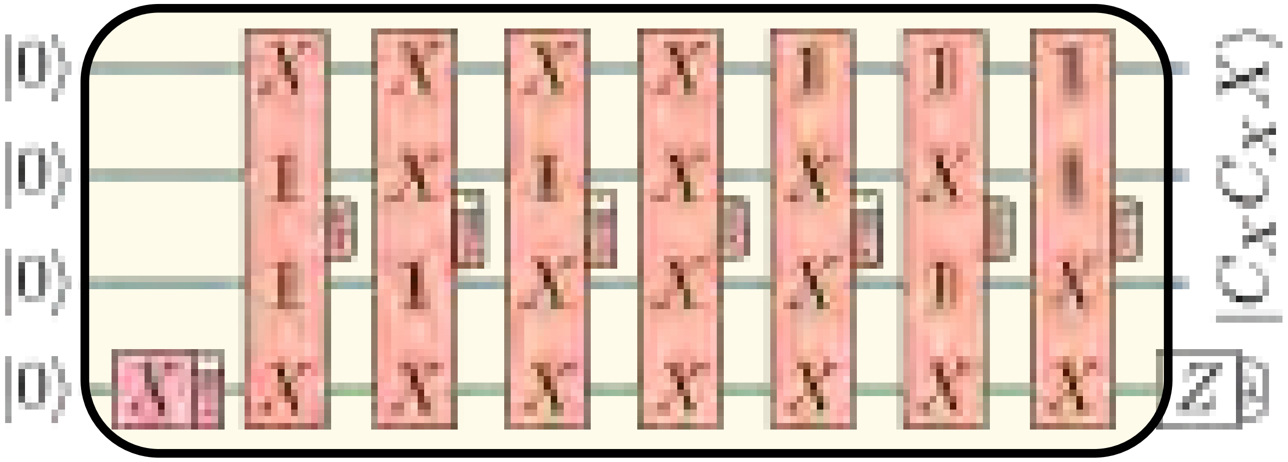
Multi qubit X rotations

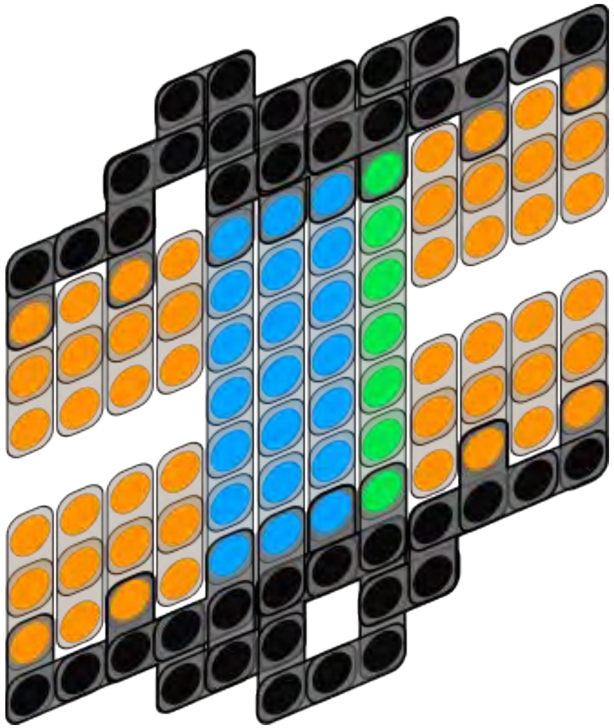




4 lattice surgery operations duration $\sim 4d_m$

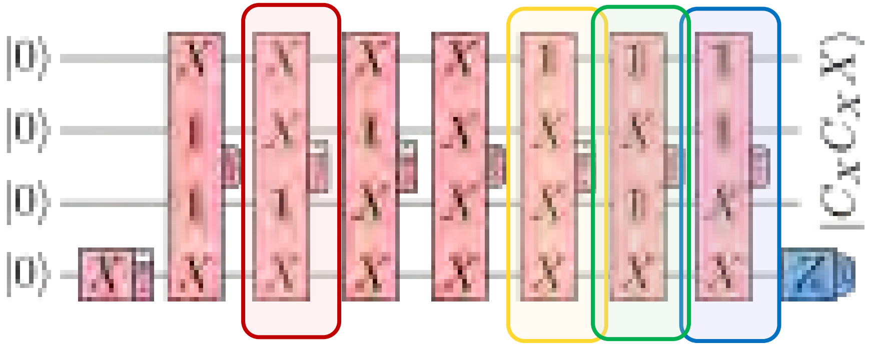
Simultaneous lattice surgery

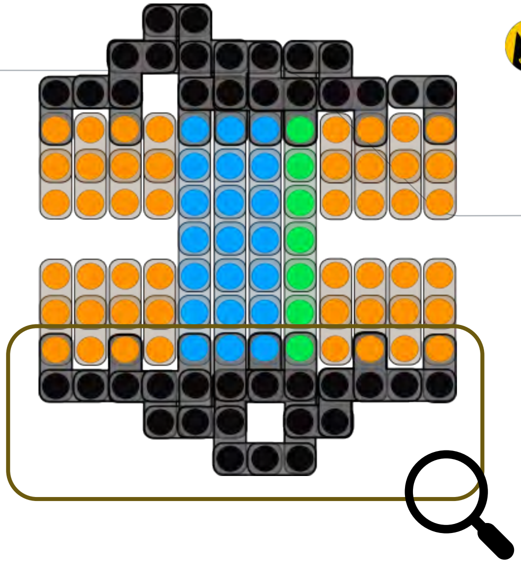


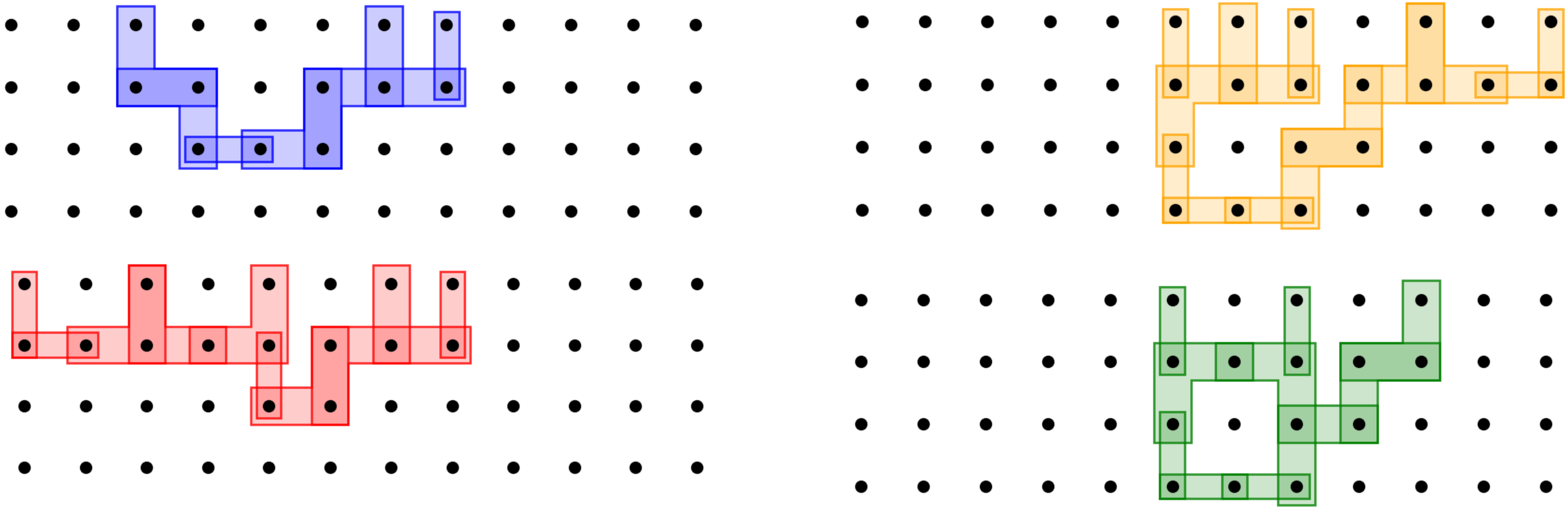


1 simultaneous surgery operation,
duration : $\sim d'_m$

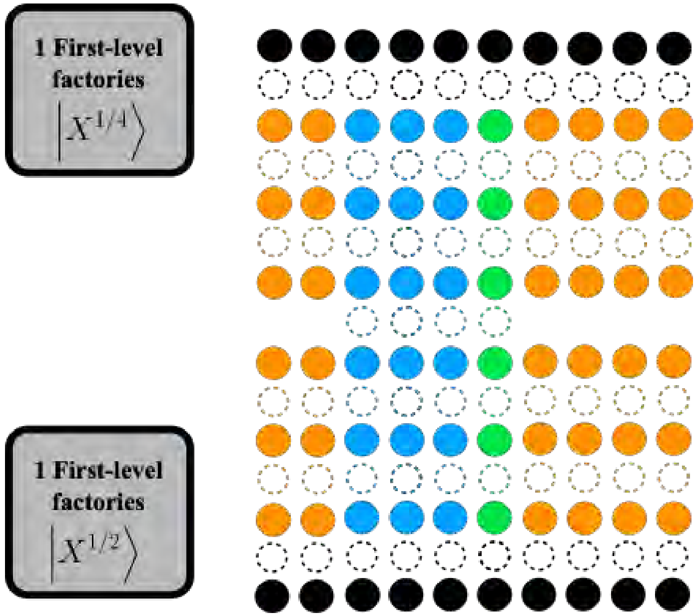
Surgery code found with **SAT solver** z3







Regular surgery



$p_{\mathrm{in},Z} = 1 \times 10^{-3}$

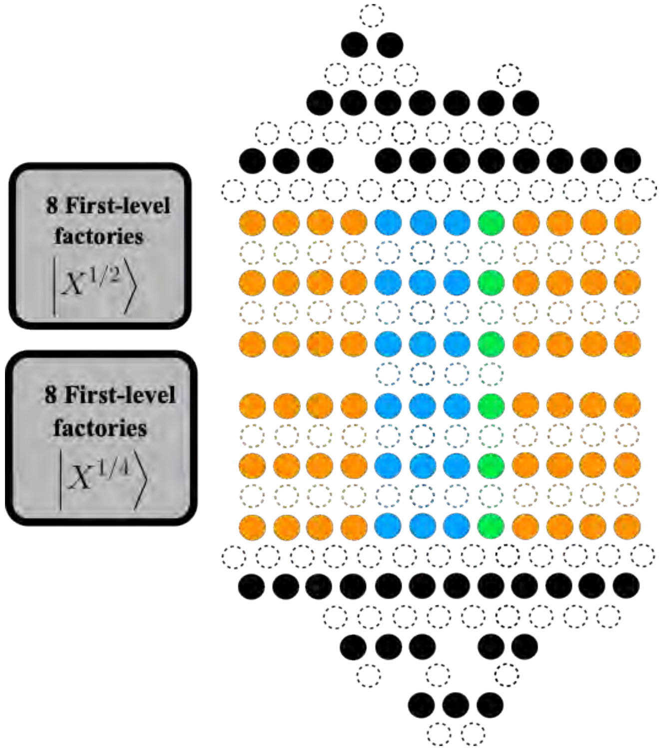
$p^{(2)}_{\mathrm{out}} = 4.7 \times 10^{-12}$

$\mathbf{4 \times 11}$ **cycles**

437 cat qubits

volume $= 19228$ cat qubits $\times$ cycles

Simultaneous surgery



$p_{\mathrm{in},Z} = 1 \times 10^{-3}$

$p^{(2)}_{\mathrm{out}} = 4.1 \times 10^{-12}$

$\mathbf{1 \times 17}$ **cycles**

864 cat qubits

volume $= 14688$ cat qubits $\times$ cycles

Summary

01. "Memory" advantage: high-density codes 4:1

Diego Ruiz *et al,* Nature Communications (2025)

02. "Compute" advantage: efficient distillation

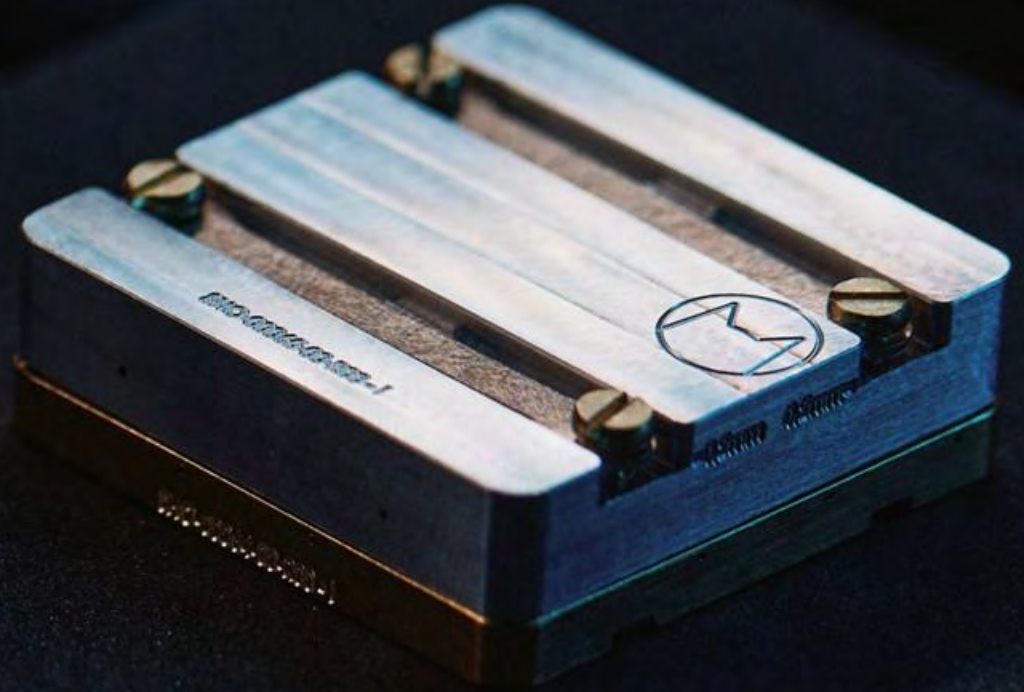
Diego Ruiz *et al*, npj Quantum information (2026)

Vivien Londe, arXiv: 2605.06284

03. "Compilation" advantage? Simultaneous lattice surgery

Hugo Jacinto et al, *in preparation*

THANK
YOU.



ALICE & BOB